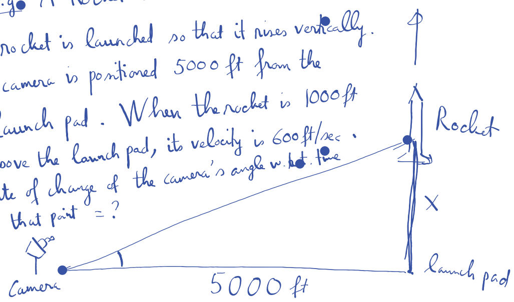
E.g. A Rocket Launch.

A rocket is launched so that it rises vertically. A camera is positioned 5000 ft from the launch pad. When the rocket is 1000 ft above the launch pad, its velocity is 600 ft/sec. Rate of change of the camera's angle w.r.t. time at that point = ?

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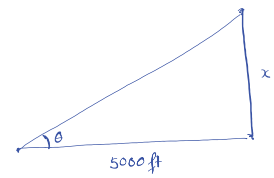
$\boxed{x}$: distance from rocket to launch pad.

$\boxed{\theta}$: camera's angle.




$\boxed{\tan\theta = \frac{x}{5000}}$ ; $\boxed{\frac{d\theta}{dt} = ?}$ ; $\boxed{\frac{dx}{dt} = 600}$, $\boxed{x = 1000}$

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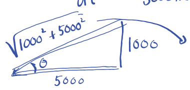
$\tan\theta = \frac{x}{5000}$

$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{x}{5000}\right)$

When $x = 1000$ ft, $\frac{dx}{dt} = 600$ ft

$\sec^2\theta \cdot \boxed{\frac{d\theta}{dt}} = \frac{1}{5000}\cdot\frac{dx}{dt}$ 600

$\frac{d\theta}{dt} = \frac{1}{5000\sec^2\theta}\cdot\boxed{\frac{dx}{dt}}$




$\rightarrow 1000\sqrt{26}$

$\cos\theta = \frac{5000}{1000\sqrt{26}} = \frac{5}{\sqrt{26}}$

$\sec\theta = \frac{\sqrt{26}}{5}$

$\sec^2\theta = \frac{26}{25}$

$\frac{d\theta}{dt} = \boxed{\frac{1}{5000\cdot\frac{26}{25}}\cdot 600} \rightarrow$ calculator. $\frac{d\theta}{dt} = \frac{3}{26}$

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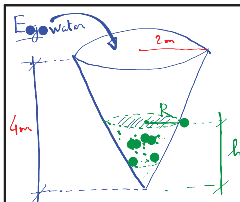
E.g. water




Water is pumped into the tank at the rate of $\boxed{2\,m^3/min}$. Find the rate at which the water level is rising when the water is 3 m deep.

Water tank (inverted circular cone)

base radius: 2m

height: 4m

(1) height of water: $\boxed{h}$

Volume of water: V

Radius: $\boxed{R}$

(2) Volume of cone $= \frac{1}{3}$ (base area)·(height)

$V = \frac{1}{3}\cdot\pi R^2\cdot h.$

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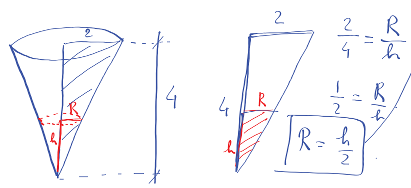
$\boxed{V = \frac{1}{3}\pi\cdot R^2\cdot h}$ (*)

$\boxed{\frac{dV}{dt} = 2.}$ Find $\boxed{\frac{dh}{dt} = ?}$ when $h = 3$.

* Solve for R in terms of h to reduce the equation to an equation with V and h only.




$\frac{2}{4} = \frac{R}{h}$

$\frac{1}{2} = \frac{R}{h}$

$\boxed{R = \frac{h}{2}}$

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$V = \frac{1}{3}\pi\cdot\left(\frac{h}{2}\right)^2\cdot h$

$V = \frac{1}{3}\pi\,\frac{h^2}{4}\cdot h = \frac{\pi}{12}\cdot h^3$

$V = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{12}\cdot 3h^2\cdot\frac{dh}{dt}.$

$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{\pi}{4}h^2}$

$\frac{dh}{dt} = \frac{2}{\frac{\pi}{4}\cdot 9} = \boxed{\frac{8}{9\pi}}$

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