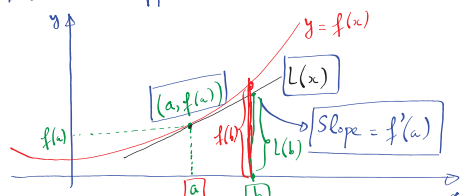


4.2 Linear approximations and Differential

* The linear approximation of a function at a point.



The linear approximation of the function $y = f(x)$ at the point $(a, f(a))$ is the function $L(x)$ whose equation is the equation of the tangent line to the graph of $y = f(x)$ at $(a, f(a))$

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$$y - f(a) = f'(a) \cdot (x - a)$$

$$y = f(a) + f'(a)(x - a)$$

$$L(x) = f(a) + f'(a)(x - a)$$

Formula for the linear approximation $L(x)$ to the function $y = f(x)$ at $x = a$

E.g. $f(x) = \sqrt{x}$. Find the linear approximation of f at $a = 9$. Use this approximation to estimate $\sqrt{9.1}$.

$$L(x) = f(a) + f'(a)(x - a) \quad \begin{matrix} f(x) = \sqrt{x} \\ a = 9 \end{matrix}$$

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$$f(x) = \sqrt{x}; \quad f'(x) = \frac{1}{2\sqrt{x}} \quad \begin{matrix} f(9) = \sqrt{9} = 3 \\ f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6} \end{matrix}$$

$$L(x) = f(9) + f'(9)(x - 9)$$

$$L(x) = 3 + \frac{1}{6}(x - 9)$$

$$L(x) = 3 + \frac{1}{6}x - \frac{3}{2}$$

$$L(x) = \frac{1}{6}x + \frac{3}{2}$$

Linear approx. to $f(x) = \sqrt{x}$ at $a = 9$.

Use this approximation to estimate $\sqrt{9.1}$

$$L(9.1) = \frac{1}{6} \cdot (9.1) + \frac{3}{2} \approx 3.0167$$

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E.g. $f(x) = e^x$. Find the linear approximation of f at the point $a = 0$. Use this approximation to estimate $e^{-0.015}$

$$L(x) = f(a) + f'(a)(x - a)$$

$$f(x) = e^x; \quad f'(x) = e^x$$

$$f(0) = e^0 = 1; \quad f'(0) = e^0 = 1$$

$$L(x) = 1 + 1 \cdot (x - 0)$$

$$L(x) = x + 1$$

$$L(-0.015) = -0.015 + 1 = 0.985$$

$$\begin{matrix} -0.015 \\ e \\ \approx 0.98511194 \end{matrix}$$

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E.g. Use a linear approximation to estimate $\sqrt[3]{1001}$

$$f(x) = \sqrt[3]{x}; \quad a = 1000$$

$$f(x) = \sqrt[3]{x}; \quad f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$

$$= \frac{1}{3x^{\frac{2}{3}}}$$

$$= \frac{1}{3(\sqrt[3]{x})^2}$$

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = 10 + \frac{1}{300}(x - 1000)$$

$$L(x) = 10 + \frac{1}{300}x - \frac{10}{3}$$

$$L(x) = \frac{1}{300}x + \frac{20}{3}$$

$$L(1001) = \frac{1}{300} \cdot 1001 + \frac{20}{3} \approx 10.00333...$$

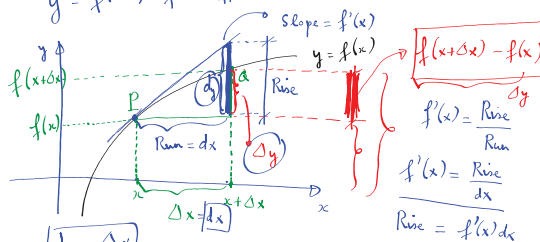
$$f(1000) = \sqrt[3]{1000} = 10$$

$$f'(1000) = \frac{1}{3 \cdot (\sqrt[3]{1000})^2}$$

$$= \frac{1}{300}$$

The differential of a function

$y = f(x)$; f : differentiable function.



The Differential dy is defined to be $dy = f'(x)dx$

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