

Note:  $dy \neq \Delta y$

$dy$  is an approximation for  $\Delta y$

When  $\Delta x$  is sufficiently small,  $dy \approx \Delta y$ .

$\Delta y$  = actual change in the function.

$$\Delta y = f(x + \Delta x) - f(x)$$

E.g.  $y = f(x) = x^3 + x^2 - 2x + 1$

Compare the values of  $[dy]$  and  $\Delta y$  when

(a)  $x$  changes from  $\boxed{2}$  to  $2.05$

$$\Delta x = \Delta x = 0.05 ; f'(x) = 3x^2 + 2x - 2$$

$$f'(2) = 3 \cdot (2)^2 + 2 \cdot 2 - 2 = 14$$

$$dy = f'(x) dx$$

$$x=2 ; dy = f'(2) \cdot (0.05)$$

$$dy = (14) \cdot (0.05) = \boxed{0.7}$$

$$\Delta y = f(x + \Delta x) - f(x) = f(2.05) - f(2) = \left( (2.05)^3 + (2.05)^2 - 2 \cdot (2.05) \right) - \boxed{9}$$

$$\Delta y = f(2.05) - f(2) = \left( (2.05)^3 + (2.05)^2 - 2 \cdot (2.05) \right) - \boxed{9} = \boxed{0.717625}$$

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Estimate error:

Suppose the side length of a cube is measured to be

$\boxed{5 \text{ cm}}$  with an accuracy of  $\boxed{0.1 \text{ cm}}$ .

\* Use differential to estimate the error in the computed volume

of the cube  $\underline{x = 5}$

$$-0.1 \leq \Delta x \leq 0.1$$



$$V = x^3 ; \quad dV = 3x^2 dx$$

$$\boxed{V = 125} \quad dV = 3 \cdot (5)^2 \cdot dx = 75 \cdot dx$$

$$\boxed{-7.5} \leq dV \leq \boxed{7.5}$$

$$\boxed{132.5}$$

$$\boxed{117.5}$$

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