4.5 Goals: (1) Use the first Derivative Test to determine the intervals of increasing / decreasing and local extrema of a function

(2) Use the second derivative test to determine the intervals of concavity of a function and inflection points.

First Derivative Test

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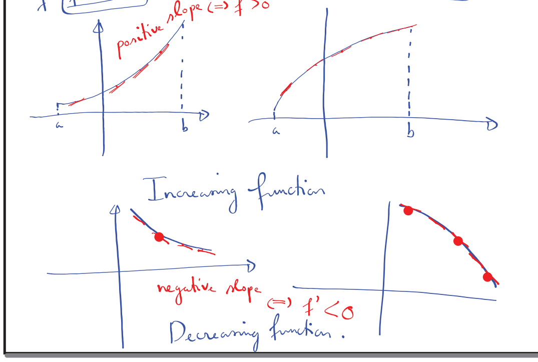
If $\boxed{f' > 0}$ on an interval I, then $\boxed{f \text{ is increasing}}$ on I

If $\boxed{f' < 0}$ on an interval I, then $\boxed{f \text{ is decreasing}}$ on I.



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* f continuous. If f has a local max/min, it must occur at a critical point — $f'(c)=0$ or $f'(c)$ DNE.

First Derivative Test

Consider a function f on an interval I.

(1) Find all the critical points of f on I. These critical points divide I into smaller subintervals.

(2) Determine the sign of f' on each subinterval

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(3) If f' changes from being >0 to being <0 at a critical # c, then f has a local max at c

$f'>0 \nearrow x=c \searrow f'<0$

If f' changes from being <0 to being >0 at a critical # c, then f has a local min at c

$f'<0 \searrow x=c \nearrow f'>0$

E.g. $f(x) = x^3 - 3x^2 - 9x - 1$.

Use the first derivative test to determine the intervals on which f is increasing/decreasing and local extrema of f.

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(1) $f'(x) = 3x^2 - 6x - 9$

$f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$\boxed{x=3}$; $\boxed{x=-1}$. ← critical points

(2)

x		$\boxed{-1}$		$\boxed{3}$	
$f'(x)$	+	0	−	0	+
$f(x)$	↗	local max @ $x=-1$	↘	local min @ $x=3$	↗

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Increasing on $(-\infty, -1) \cup (3, \infty)$

Decreasing on $(-1, 3)$

local max @ $\boxed{x = -1}$. local max value is

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 1$

$= -1 - 3 + 9 - 1$

$= \boxed{4}$

local min @ $x = 3$. local min value is

$f(3) = (3)^3 - 3\cdot(3)^2 - 9(3) - 1$

$= 27 - 27 - 27 - 1$

$= \boxed{-28}$

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