E.g. $f(x) = 5x^{1/3} - x^{5/3}$. Domain $(-\infty, \infty)$.

Use the first derivative test to determine intervals of increasing / decreasing & local max/min of f.

(1) $f'(x) = \frac{5}{3}x^{-2/3} - \frac{5}{3}x^{2/3}$

$$= \frac{5}{3x^{2/3}} - \frac{5x^{2/3}}{3\cdot x^{2/3}} = \frac{5 - 5x^{4/3}}{3x^{2/3}} = \boxed{\frac{5(1-x^{4/3})}{3x^{4/3}}}$$

* f' is undefined when $3x^{2/3} = 0$; $\boxed{x=0}$
* $f' = 0$ when $5(1 - x^{4/3}) = 0$

$x^{4/3} = 1$

$\sqrt[3]{x^4} = 1$; $x^4 = 1$; $\boxed{x = \pm 1}$

Oct 24-4:51 PM

Critical points $x=0$; $x=-1$; $x=1$

(2)

x	test point -2	$\boxed{-1}$	test point $-\frac{1}{2}$	$\boxed{0}$	test point $\frac{1}{2}$	$\boxed{1}$	test point 2
$f'(x)$	$-$	0	$+$	0	$+$	0	$-$
$f(x)$		local min @ $x=-1$		neither max nor min		local max @ $x=1$	

Increasing on $(-1, 0) \cup (0, 1)$

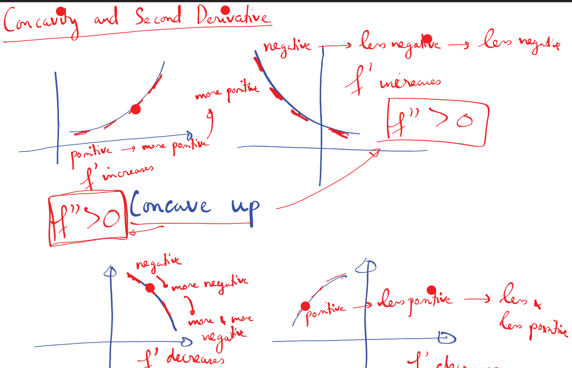
Decreasing on $(-\infty, -1) \cup (1, \infty)$

local min @ $x = -1$. Min value $f(-1) = \boxed{-4}$

Local max @ $x = 1$; Max value $f(1) = \boxed{4}$

Oct 24-4:56 PM

Concavity and Second Derivative



$\boxed{f''>0}$ Concave up

$\boxed{f''<0}$ Concave down $\boxed{f''<0}$

Oct 24-5:01 PM

$f'' < 0 \implies f$ is concave down $\cap$

$f'' > 0 \implies f$ is concave up $\cup$

$\boxed{f''<0}$

$\boxed{f''>0}$

$f''(c) = 0$ (<0)

Note: If f'' changes from being >0 to being <0 at a pt c (>0) we call c an inflection point of f.

Oct 24-5:06 PM

E.g. $f(x) = -x^3 + \frac{3}{2}x^2 + 18x$

Use the second derivative to determine the intervals of concavity and inflection points of f.

$f'(x) = -3x^2 + 3x + 18$

$f''(x) = \boxed{-6x+3} = 0$

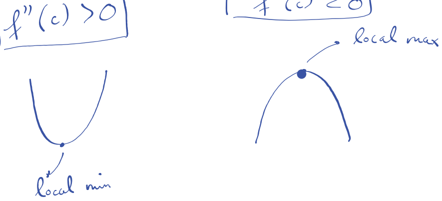
$-6x = -3$

$x = \frac{1}{2}$

x		$\boxed{\frac{1}{2}}$	
$f''(x)$	$+$	0	$-$
f	$\cup$	inflection point @ $x = 1/2$	$\cap$

Oct 24-5:09 PM

$\boxed{\begin{array}{l} f'(c) = 0 \\ f''(c) > 0 \end{array}}$ $\boxed{\begin{array}{l} f'(c) = 0 \\ f''(c) < 0 \end{array}}$



Oct 24-5:12 PM