

E.g. $f(x) = 5x^{1/3} - x^{5/3}$. Domain $(-\infty, \infty)$.

Use the first derivative test to determine interval of increasing/decreasing a local max/min of f .

(1) $f'(x) = \frac{5}{3}x^{-2/3} - \frac{5}{3}x^{2/3}$

$$= \frac{5}{3x^{2/3}} - \frac{5x^{2/3} \cdot x^{1/3}}{3 \cdot x^{2/3}} = \frac{5 - 5x^{4/3}}{3x^{2/3}} = \frac{5(1 - x^{4/3})}{3x^{2/3}}$$

* f' is undefined when $3x^{2/3} = 0$, $x = 0$

* $f' = 0$ when $5(1 - x^{4/3}) = 0$

$$x^{4/3} = 1$$
$$\sqrt[3]{x^4} = 1, x^4 = 1, x = \pm 1$$

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Critical points $x=0$, $x=-1$; $x=1$

②

x	test point -2	-1	test point $-\frac{1}{2}$	0	test point $\frac{1}{2}$	1	test point 2
$f'(x)$	$-$	0	$+$	0	$+$	0	$-$
$f(x)$		local min @ $x=-1$		neither max nor min		local max @ $x=1$	

Increasing on $(-1, 0) \cup (0, 1)$
Decreasing on $(-\infty, -1) \cup (1, \infty)$
local min @ $x = -1$ • Min value $f(-1) = [-4]$
local max @ $x = 1$ • Max value $f(1) = [4]$

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Concavity and Second Derivative

Graph 1 (Top Left): A curve is shown in the first quadrant, increasing and concave up. A red dot is marked on the curve. Arrows indicate the curve is becoming "more positive" and "more positive" as it moves right. A box contains $f'' > 0$ and the text "Concave up".

Graph 2 (Top Right): A curve is shown in the second quadrant, increasing and concave down. A red dot is marked on the curve. Arrows indicate the curve is becoming "less negative" and "less negative" as it moves right. A box contains $f'' > 0$ and the text "Concave up".

Graph 3 (Bottom Left): A curve is shown in the third quadrant, decreasing and concave down. A red dot is marked on the curve. Arrows indicate the curve is becoming "more negative" and "more & more negative" as it moves right. A box contains $f'' < 0$ and the text "Concave down".

Graph 4 (Bottom Right): A curve is shown in the fourth quadrant, decreasing and concave up. A red dot is marked on the curve. Arrows indicate the curve is becoming "less positive" and "less & less positive" as it moves right. A box contains $f'' < 0$ and the text "Concave down".

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$f'' < 0 \Rightarrow f$ is concave down 
 $f'' > 0 \Rightarrow f$ is concave up 
 $f'' < 0$ 
 $f'' > 0$ 
 $f'(c) = 0$ (< 0)
Note: If f' changes from being > 0 to being < 0 at a pt c (> 0) we call c an inflection point of f .

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Use the second derivative to determine the intervals of concavity and inflection points of f .

$$f'(x) = -3x^2 + 3x + 18$$

$$f''(x) = -6x + 3 = 0$$

$$-6x = -3$$

$$x = \frac{1}{2}$$

x	$\frac{1}{2}$
$f''(x)$	$+$ 0 $-$
f	$\cup$ $\cap$

inflection point $x = \frac{1}{2}$

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Left side (Local Minimum):

$$\begin{aligned} f'(c) &= 0 \\ f''(c) &> 0 \end{aligned}$$

local min

Right side (Local Maximum):

$$\begin{aligned} f'(c) &= 0 \\ f''(c) &< 0 \end{aligned}$$

local max

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