4.6 Limits at Infinity and Asymptotes

Goals (1) Compute limits at infinity and use this idea to find the horizontal asymptotes of functions.

(2) Put everything together to graph functions.

Limits at ∞

$f(x)$: function of x.

Behavior of f when x is very large positively? $\lim_{x\to\infty} f(x)$

Behavior of f when x is very large negatively? $\lim_{x\to-\infty} f(x)$

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Basic limits at ∞

$f(x) = x$; $\lim_{x\to\infty} x = \infty$, $\lim_{x\to-\infty} x = -\infty$

$f(x) = x^2$; $\lim_{x\to\infty} x^2 = \infty$, $\lim_{x\to-\infty} x^2 = \infty$

$f(x) = x^3$; $\lim_{x\to\infty} x^3 = \infty$; $\lim_{x\to-\infty} x^3 = -\infty$

$f(x) = x^4$; $\lim_{x\to\infty} x^4 = \infty$; $\lim_{x\to-\infty} x^4 = \infty$

(I) $\lim_{x\to\infty} x^n = \infty$, n is any positive integer

$\lim_{x\to-\infty} x^n = \begin{cases} \infty & \text{if } n \text{ is an even positive integer} \\ -\infty & \text{if } n \text{ is an odd positive integer} \end{cases}$

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$f(x) = \frac{1}{x}$; $\lim_{x\to\infty} \frac{1}{x} = 0$, $\lim_{x\to-\infty} \frac{1}{x} = 0$

$f(x) = \frac{1}{x^2}$; $\lim_{x\to\infty} \frac{1}{x^2} = 0$, $\lim_{x\to-\infty} \frac{1}{x^2} = 0$

$\lim_{x\to\infty} \frac{1}{x^n} = 0$, $\lim_{x\to-\infty} \frac{1}{x^n} = 0$

$\lim_{x\to\pm\infty} \frac{1}{x^n} = 0$ (II)

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$f(x) = e^x$

$\lim_{x\to\infty} e^x = \infty$, $\lim_{x\to-\infty} e^x = 0$

$\lim_{x\to\infty} e^{-x} = 0$, $\lim_{x\to-\infty} e^{-x} = \infty$

$\lim_{x\to\infty} \ln(x) = \infty$, $\lim_{x\to-\infty} \ln(x)$ DNE

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$f(x) = \tan^{-1}(x)$ or $\arctan(x)$

$\lim_{x\to\infty} (\tan^{-1}(x)) = \frac{\pi}{2}$

$\lim_{x\to-\infty} \tan^{-1}(x) = -\frac{\pi}{2}$

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E.g. (1) $\lim_{x\to\infty} \left(3 + \frac{1}{x}\right) = \lim_{x\to\infty} 3 + \lim_{x\to\infty} \frac{1}{x} = 3 + 0 = 3$

$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to\infty} 3 = 3$

(2) $\lim_{x\to\infty} \frac{1}{2x+5} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{2x+5}{x}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{2+\frac{5}{x}}$ ($\frac{1}{x} \to 0$, $\frac{5}{x} \to 0$)

$= \frac{0}{2+0} = \frac{0}{2} = 0$

$\lim_{x\to\infty} \frac{1}{2x+5} = 0$

2nd way to see why the limit is 0: $\lim_{x\to\infty} \frac{1}{2x+5}$

$\frac{1}{2x+5} \sim \frac{1}{2x} \to 0$ (when x is large; insignificant term)

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