

4.6 Limits at Infinity and Asymptotes

- Goals
- ① Compute limits at infinity and use this idea to find the horizontal asymptotes of functions.
 - ② Put everything together to graph functions.

Limits at ∞

$f(x)$: function of x .

Behavior of f when x is very large positively?

$$\lim_{x \rightarrow \infty} f(x)$$

Behavior of f when x is very large negatively? $\lim_{x \rightarrow -\infty} f(x)$

Basic limits at ∞

$$f(x) = x; \lim_{x \rightarrow \infty} x = \infty$$

$$\lim_{x \rightarrow -\infty} x = -\infty$$

$$f(x) = x^2; \lim_{x \rightarrow \infty} x^2 = \infty$$

$$\lim_{x \rightarrow -\infty} x^2 = \infty$$

$$f(x) = x^3; \lim_{x \rightarrow \infty} x^3 = \infty; \lim_{x \rightarrow -\infty} x^3 = -\infty$$

$$f(x) = x^4; \lim_{x \rightarrow \infty} x^4 = \infty; \lim_{x \rightarrow -\infty} x^4 = \infty$$

$$\textcircled{I} \quad \lim_{x \rightarrow \infty} x^n = \infty; n \text{ is any positive integer}$$

$$\lim_{x \rightarrow -\infty} x^n = \begin{cases} \infty & \text{if } n \text{ is an even positive integer} \\ -\infty & \text{if } n \text{ is an odd positive integer} \end{cases}$$

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$$f(x) = \frac{1}{x}; \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{1}{x} = 0$$

$$f(x) = \frac{1}{x^2}; \lim_{x \rightarrow \infty} \frac{1}{x^2} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{1}{x^2} = 0$$

$$\lim_{x \rightarrow \infty} \frac{1}{x^n} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{1}{x^n} = 0$$

$$\lim_{x \rightarrow \pm\infty} \frac{1}{x^n} = 0 \quad \textcircled{II}$$

$$f(x) = e^x$$

$$\lim_{x \rightarrow \infty} e^x = \infty$$

$$\lim_{x \rightarrow -\infty} e^x = 0$$

$$\lim_{x \rightarrow \infty} e^{-x} = 0$$

$$\lim_{x \rightarrow -\infty} e^{-x} = \infty$$

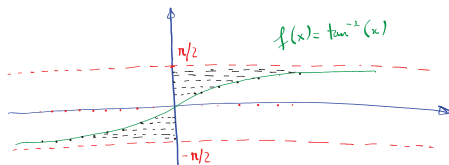
$$\lim_{x \rightarrow \infty} \ln(x) = \infty$$

$$\lim_{x \rightarrow -\infty} \ln(x)$$

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$$f(x) = \tan^{-1}(x) \text{ or } \arctan(x)$$



$$\lim_{x \rightarrow \infty} \tan^{-1}(x) = \frac{\pi}{2}$$

$$\lim_{x \rightarrow -\infty} \tan^{-1}(x) = -\frac{\pi}{2}$$

$$\text{E.g. } \textcircled{1} \quad \lim_{x \rightarrow \infty} \left(3 + \frac{1}{x} \right) = \lim_{x \rightarrow \infty} 3 + \lim_{x \rightarrow \infty} \frac{1}{x} = 3 + 0 = 3$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$\lim_{x \rightarrow \infty} 3 = 3$$

$$\textcircled{2} \quad \lim_{x \rightarrow \infty} \frac{1}{2x+5} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{2x+5}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{2 + \frac{5}{x}} = \frac{0}{2+0} = \frac{0}{2} = 0$$

$$\lim_{x \rightarrow \infty} \frac{1}{2x+5} = 0$$

$$\text{2nd way to see why the limit is 0: } \lim_{x \rightarrow \infty} \frac{1}{2x+5}$$

$\frac{1}{2x+5} \approx \frac{1}{2x}$ (when x is large) insignificant term

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