

$$\lim_{x \rightarrow \infty} \frac{10x^2 - 2x + 5}{13x^2 + 7} \rightarrow \text{degree 2} / \text{degree 2}$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - 2x + 5}{3x^2 + 7} = \lim_{x \rightarrow \infty} \frac{1 - \frac{2}{x} + \frac{5}{x^2}}{3 + \frac{7}{x^2}} = \frac{1}{3}$$

2<sup>nd</sup> way to find this limit

$$\frac{x^2 - 2x + 5}{3x^2 + 7} \approx \frac{x^2}{3x^2} = \frac{1}{3} \text{ (when } x \text{ is large)}$$

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$$\lim_{x \rightarrow \infty} \frac{x^2 - 2x + 5}{3x + 7} \rightarrow \text{degree 2} / \text{degree 1} \approx \frac{x^2}{3x} = \frac{x}{3} \rightarrow \infty \text{ (when } x \text{ is large)}$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - 2x + 5}{2x + 7} = \infty$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - 2x + 5}{2x + 7} \approx \frac{x^2}{2x} = \frac{x}{2} \rightarrow \infty$$

$$\lim_{x \rightarrow \infty} \frac{3x + 7}{x^2 - 2x + 5} \rightarrow \text{degree 1} / \text{degree 2} \approx \frac{3x}{x^2} = \frac{3}{x} \rightarrow 0$$

In general, for rational functions (functions of the form  $\frac{p(x)}{q(x)}$  where  $p$  and  $q$  are polynomials.)

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\* 1<sup>st</sup> case: Degree  $p$  = Degree  $q$

$$\lim_{x \rightarrow \infty} \frac{p(x)}{q(x)} = \frac{a}{b}; \quad a: \text{leading coefficient of } p(x), \quad b: \text{leading coefficient of } q(x)$$

$$\lim_{x \rightarrow \infty} \frac{p(x)}{q(x)} = \frac{a}{b}$$

\* 2<sup>nd</sup> case: Degree  $p$  < Degree of  $q$ .

$$\lim_{x \rightarrow \pm \infty} \frac{p(x)}{q(x)} = 0$$

\* 3<sup>rd</sup> case: Degree  $p$  > Degree  $q$

$$\lim_{x \rightarrow \pm \infty} \frac{p(x)}{q(x)} = \begin{cases} \infty \\ -\infty \end{cases}$$

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E.g.  $\lim_{x \rightarrow \infty} \frac{3x - 2}{\sqrt{4x^2 + 5}} \approx \frac{3x}{\sqrt{4x^2}} = \frac{3x}{2\sqrt{x^2}} = \frac{3x}{2x} = \frac{3}{2}$

$$\lim_{x \rightarrow -\infty} \frac{3x - 2}{\sqrt{4x^2 + 5}} \approx \frac{3x}{\sqrt{4x^2}} = \frac{3x}{2\sqrt{x^2}} = \frac{3x}{2(-x)} = -\frac{3}{2}$$

$\sqrt{x^2} = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$  Kind of like  $|x|$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

E.g.  $\lim_{x \rightarrow \infty} \frac{1 - e^x}{1 + 2e^x} \approx \frac{-e^x}{2e^x} = -\frac{1}{2}$

Prove it:

$$\lim_{x \rightarrow \infty} \frac{1 - e^x}{1 + 2e^x} = \lim_{x \rightarrow \infty} \frac{\frac{1}{e^x} - 1}{\frac{1}{e^x} + 2} = \lim_{x \rightarrow \infty} \frac{\frac{e^{-x}}{e^x} - 1}{\frac{e^{-x}}{e^x} + 2} = \frac{-1}{2}$$

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$$\lim_{x \rightarrow \infty} \arctan(e^x) = \frac{\pi}{2}$$

More

Precise way: let  $u = e^x$ . When  $x \rightarrow \infty$ ,  $u \rightarrow \infty$

Original limit becomes  $\lim_{u \rightarrow \infty} \arctan(u) = \frac{\pi}{2}$

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