4.10. Antiderivatives

(1) Find the general antiderivative of certain functions

(2) Solve some basic initial value problems

Definition of an antiderivative of a function.

f : function defined on an interval I.

A function F is an antiderivative of f over I if

$$F'(x) = f(x) \text{ for every } x \text{ in } I.$$

E.g. $f(x) = x$ defined over $(-\infty, \infty)$

$\boxed{F(x) = \frac{x^2}{2}}$. Then $F'(x) = \left(\frac{x^2}{2}\right)' = x = f(x)$

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$F(x) = \frac{x^2}{2}$ is an antiderivative of $f(x) = x$.

$G(x) = \frac{x^2}{2} + 3$. Then $G'(x) = \left(\frac{x^2}{2} + 3\right)' = x = f(x)$

$H(x) = \frac{x^2}{2} + \pi$. Then $H'(x) = f(x)$

Any function of the form $\boxed{F(x) = \frac{x^2}{2} + C}$, where C is a constant is an antiderivative of $f(x) = x$

Theorem: If $F(x)$ and $G(x)$ are 2 antiderivatives of $f(x)$ (i.e. $F'(x) = G'(x) = f(x)$), then $F(x) = G(x) + C$.

Therefore if $F(x)$ is an antiderivative of $f(x)$, then the most general antiderivative of $f(x)$ has the form $\boxed{F(x) + C}$, C : constant

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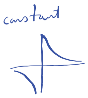
E.g. Find the general antiderivative of the given function.

(1) $f(x) = x^2$.
$F(x) = \frac{x^3}{3} + C$

(2) $f(x) = x^3$
$F(x) = \frac{x^4}{4} + C$.

In general, if $f(x) = x^n$; $n \neq -1$, then the general antiderivative of $f(x)$ is $\boxed{F(x) = \frac{x^{n+1}}{n+1} + C}$, C : constant

What if $n = -1$? $f(x) = x^{-1} = \frac{1}{x}$.



$\boxed{F(x) = \ln|x| + C}$

$\left(\text{Recall: } (\ln|x|)' = \frac{1}{x}\right)$

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$f(x) = \cos x$; $F(x) = \sin x + C$.

$f(x) = \sin x$; $F(x) = -\cos x + C$.

Important Notation: Indefinite Integral Notation.

$$\boxed{\int} \boxed{f(x)} \boxed{dx} := \text{the general antiderivative of } f(x)$$

variable of integration

integrand

indefinite integral notation

E.g. $\int x^2\,dx = \frac{x^3}{3} + C$

$\int \sin x\,dx = -\cos x + C$.

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Table of useful indefinite integrals

Function	Indefinite Integral
$f(x) = x^n$; $n \neq -1$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$; $n \neq -1$
$f(x) = 1$	$\int 1\,dx = \int dx = x + C$
$f(x) = k$; k : constant	$\boxed{\int k\,dx = kx + C}$
$f(x) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln|x| + C$
$f(x) = e^x$	$\int e^x\,dx = e^x + C$
$f(x) = \sin x$	$\int \sin x\,dx = -\cos x + C$
$f(x) = \cos x$	$\int \cos x\,dx = \sin x + C$
$f(x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$

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$f(x) = \csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
$f(x) = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$f(x) = \csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$.
$f(x) = \frac{1}{1+x^2}$	$\boxed{\int \frac{1}{1+x^2}\,dx = \arctan x + C}$
$f(x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$.

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