Properties of Indefinite Integrals

$$\boxed{\int \left(f(x) \pm g(x)\right) dx = \int f(x)\,dx \pm \int g(x)\,dx}$$

Note: We DO NOT have

$$\cancel{\int f(x)\,g(x)\,dx = \int f(x)\,dx \int g(x)\,dx}$$

$$\cancel{\int \frac{f(x)}{g(x)}\,dx = \frac{\int f(x)\,dx}{\int g(x)\,dx}}$$

$$\boxed{\int c\cdot f(x)\,dx = c\cdot\int f(x)\,dx}$$

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E.g. of finding indefinite integrals.

(1) $\int \left(7x^{2/5} + 8x^{-4/5}\right) dx$

$= \int 7x^{2/5}\,dx + \int 8x^{-4/5}\,dx = 7\int x^{\boxed{2/5}}\,dx + 8\int x^{\boxed{-4/5}}\,dx$

$= 7\cdot\frac{x^{\frac{2}{5}+1}}{\frac{2}{5}+1} + 8\cdot\frac{x^{-\frac{4}{5}+1}}{-\frac{4}{5}+1} = 7\cdot\frac{x^{\frac{7}{5}}}{\frac{7}{5}} + 8\cdot\frac{x^{\frac{1}{5}}}{\frac{1}{5}}$

$= 7\cdot\frac{5}{7}\cdot x^{\frac{7}{5}} + 8\cdot 5\cdot x^{\frac{1}{5}} = 5x^{\frac{7}{5}} + 40x^{\frac{1}{5}} + C.$

(2) $\int \left(2\sin x - \sec^2 x\right) dx = \int 2\sin x\,dx - \int \sec^2 x\,dx$

$= 2\int \sin x\,dx - \int \sec^2 x\,dx$

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$= 2(-\cos x) + C - \tan x = -2\cos x - \tan x + C.$

(3) $\int \boxed{e^2}\,dx = e^2 x + C$

constant

(4) $\int \left(4\sqrt{x} + \sqrt[4]{x}\right) dx$

$= \int \left(4x^{\frac{1}{2}} + x^{\frac{1}{4}}\right) dx = 4\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1}$

$= 4\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{5}{4}}}{\frac{5}{4}} = \frac{8}{3}x^{\frac{3}{2}} + \frac{4}{5}x^{\frac{5}{4}} + C.$

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(5) $\int (x+1)(2x-1)\,dx \neq \cancel{\int (x+1)\,dx \cdot \int (2x-1)\,dx}$

$\int \left(2x^2 - x + 2x - 1\right) dx = \int \left(2x^2 + x - 1\right) dx$

$= \boxed{2\cdot\frac{x^3}{3} + \frac{x^2}{2} - x + C}$

(6) $\int \frac{2x^5 - \sqrt{x}}{x}\,dx = \int \left(\frac{2x^5}{x} - \frac{\sqrt{x}}{x}\right) dx$

$= \int \left(2x^4 - \frac{x^{\frac{1}{2}}}{x^1}\right) dx = \int \left(2x^4 - x^{\frac{1}{2}-1}\right) dx = \int \left(2x^4 - x^{-\frac{1}{2}}\right) dx$

$= 2\cdot\frac{x^5}{5} - \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{2}{5}x^5 - 2\cdot x^{\frac{1}{2}} = \frac{2}{5}x^5 - 2\sqrt{x} + C$

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(7) $\int \frac{2+x^2}{1+x^2}\,dx$

$\int \frac{(1)+(1+x^2)}{1+x^2}\,dx = \int \left(\frac{1}{1+x^2} + 1\right) dx$

$= \arctan x + x + C$

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Basic Initial Value Problems

E.g. Find the function $f(x)$ given that

$f'(x) = \frac{2}{x^2} - \frac{x^2}{2}$

$\boxed{f(1) = 0}$ ← allows to find C

$f(x) = \int \left(\frac{2}{x^2} - \frac{x^2}{2}\right) dx = \int \left(2x^{-2} - \frac{x^2}{2}\right) dx$

$= 2\cdot\frac{x^{-1}}{-1} - \frac{x^3}{6} + C = -\frac{2}{x} - \frac{x^3}{6} + C$

$f(x) = -\frac{2}{x} - \frac{x^3}{6} + C.$

Since $f(1) = 0$; $-\frac{2}{1} - \frac{1}{6} + C = 0$

$-\frac{13}{6} + C = 0$

$\boxed{C = \frac{13}{6}}$

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