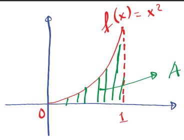

A = precise area under $f(x) = x^2$ from $x = 0$ to $x = 1$

$$0.21875 = \frac{7}{32} < A < \frac{15}{32} = 0.46875$$

How to get better approximation $\longrightarrow$ use more rectangles.

* $n = 8$ rectangles: $L_8 = 0.2734$
  $R_8 = 0.3984$

$$0.2734 < A < 0.3984$$

* $n = 50$ rectangles $L_{50} = 0.3234$; $R_{50} = 0.3434$

$$0.\boxed{3}234 < A < 0.\boxed{3}434 \implies A \approx 0.3\ldots$$

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$n = 75$; $L_{75} = 0.3266$
$R_{75} = 0.3400$

$$0.3266.. < A < 0.3400\ldots$$

$n = 100$; $L_{100} = 0.32835$; $R_{100} = 0.33835$

$$0.3\boxed{2}835 < A < 0.3\boxed{3}835$$

Increase n $\longrightarrow$ $A \approx 0.33\ldots$

n	L_n <	A < R_n
4	0.21875	0.46875
8	0.2734	0.3984
50	0.3234	0.3434
100	0.32835	0.33835

It has been proved that if f is continuous

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n$$

and these limits = precise area under the curve f.

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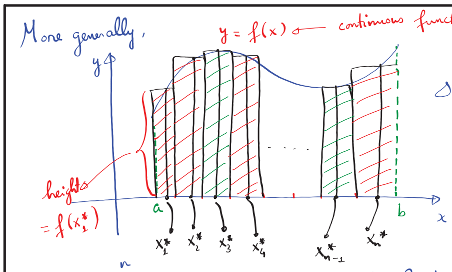
More generally,



$$\Delta x = \frac{b-a}{n}$$

$$\lim_{n\to\infty} \underbrace{\sum_{i=1}^{n} \underbrace{f(x_i^*)}_{\text{height of } i^{\text{th}} \text{ rectangle}} \cdot \underbrace{\Delta x}_{\text{width}}}_{\text{area of } i^{\text{th}} \text{ rectangle}}$$

$\longrightarrow$ this limit always exists, this limit gives us the precise area under the curve f from $x = a$ to $x = b$

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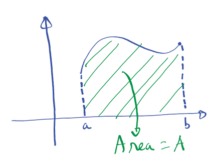
The definite integral is defined to be

$$\int_a^b f(x)\,dx := \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

<u>Note</u>: if $f \geq 0$, $\int_a^b f(x)\,dx$ = Area under f from $x = a$ to $x = b$



$$\boxed{\int_a^b f(x)\,dx = A}$$

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