

$f(x) = \frac{x}{4+x}$; $[0, 100]$

Find the absolute max/min.

① Critical Points. $f'(x) = \frac{1(4+x) - 1 \cdot x}{(4+x)^2}$
 $f'(x) = \frac{4+x-x}{(4+x)^2} = \frac{4}{(4+x)^2}$

No critical point.

② $f(0) = 0$; $f(100) = \frac{100}{104}$
 abs. min (0, 0) abs. max $(100, \frac{25}{26})$

Oct 14-12:41 PM

$f(x) = \frac{x^2-1}{x^2+8x-9}$ Domain: $x^2+8x-9=0$
 $(x-1)(x+9)=0$ $x=1, x=-9$

Critical Points in the domain. Domain: $(-\infty, -9) \cup (-9, 1) \cup (1, \infty)$

$f(x) = \frac{(x-1)(x+1)}{(x-1)(x+9)} = \frac{x+1}{x+9}$

$f'(x) = \frac{1(x+9) - (x+1)}{(x+9)^2} = \frac{x+9-x-1}{(x+9)^2} = \frac{8}{(x+9)^2}$

Oct 14-12:49 PM

$\frac{1}{2} = \frac{R}{h}$
 $2R = h$
 $R = \frac{h}{2}$

① Radius: R
 Height: h
 Volume: V

② $\frac{dV}{dt} = -0.03$
 $\frac{dh}{dt} = ?$

③ $V = \frac{1}{3}(\text{base area}) \cdot (\text{height})$
 $V = \frac{1}{3} \pi R^2 \cdot h$ Solve for R in terms of h

0.03 ft³/s

2 ft

Oct 14-12:58 PM

$V = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 \cdot h = \frac{1}{3} \pi \cdot \frac{h^2}{4} \cdot h = \frac{\pi}{12} h^3$

$V = \frac{\pi}{12} h^3$

$-0.03 = \frac{\pi}{12} \cdot 3 \cdot \frac{1}{4} \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3 \cdot \frac{1}{4} \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-0.03}{\frac{\pi}{12} \cdot \frac{3}{4}}$

$\approx$

Oct 14-1:07 PM

$f(1) = 10$

$f'(x) \geq 2$ for every x with $1 \leq x \leq 4$

What is the smallest value of $f(4)$?

$[1, 4]$ MVT:
 there is a number c in $(1, 4)$ such that

$f'(c) = \frac{f(4) - f(1)}{4 - 1}$

$\frac{f(4) - 10}{3} = f'(c) \geq 2$

Oct 14-1:14 PM

$\frac{f(4) - 10}{3} \geq 2$

$f(4) - 10 \geq 6$

$f(4) \geq 16$

#7 $f(x) = x + \sin x = 0$
 $f(0) = 0$
 f is one-to-one on $(-1, 1)$
 $f^{-1}(0) = 0$

Find $(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(0)} = \frac{1}{1 + \cos 0} = \frac{1}{2}$

$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$

Oct 14-1:17 PM