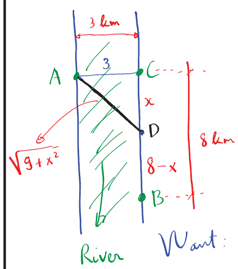
Example of Optimization Problem Similar to #24



A man launches his boat from pt A and want reach B.

He can row $\boxed{6 \text{ km/h}}$

He can run $\boxed{8 \text{ km/h}}$

Where should he land to reach B in the shortest amount of time?

Want: minimize time. $\boxed{T}$

Time it takes to row from A to D: $\frac{AD}{6} = \frac{\sqrt{9+x^2}}{6}$

Time it takes to run from D to B: $\frac{DB}{8} = \frac{8-x}{8}$

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$$\boxed{T(x) = \frac{\sqrt{9+x^2}}{6} + \frac{8-x}{8}}$$

Goal: minimize $T(x)$. Restrictions on x: $\boxed{0 \le x \le 8}$

$$T'(x) = \frac{1}{6} \cdot \frac{\cancel{2}x}{\cancel{2}\sqrt{9+x^2}} - \frac{1}{8}$$

$$T'(x) = 0; \quad \frac{x}{6\sqrt{9+x^2}} - \frac{1}{8} = 0$$

$$\frac{x}{6\sqrt{9+x^2}} = \frac{1}{8}$$

$$x = \frac{6}{8}\sqrt{9+x^2} = \frac{3}{4}\sqrt{9+x^2}$$

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$$(x)^2 = \left(\frac{3}{4}\sqrt{9+x^2}\right)^2; \quad x^2 = \frac{9}{16}(9+x^2);$$

$$x^2 = \frac{81}{16} + \frac{9}{16}x^2; \quad \frac{7}{16}x^2 = \frac{81}{16}; \quad x^2 = \frac{81}{7}$$

$x = \pm \frac{9}{\sqrt{7}}$; Since $x \ge 0$, $\boxed{x = \frac{9}{\sqrt{7}}}$ ← critical pt

$T(0) = \frac{3}{2} = 1.5$ $\quad T\left(\frac{9}{\sqrt{7}}\right) = \boxed{\frac{\sqrt{9+\frac{81}{7}}}{6} + \frac{8-\frac{9}{\sqrt{7}}}{8}} \approx 1.33$

$T(8) = \frac{\sqrt{73}}{6} \approx 1.42$. Conclusion $x = \frac{9}{\sqrt{7}}$ minimizes T. Min $T = 1.33$ h.

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HW 5.3-6 FTC, part 1

$$\frac{d}{dx}\left(\int_1^{e^x} \ln(u^5)\, du\right) = \ln\left((e^x)^5\right) \cdot e^x$$

FTC part 1

$$= \ln(e^{5x}) \cdot e^x = 5x \cdot \underbrace{\ln(e)}_{1} \cdot e^x$$

$$= \boxed{5x \cdot e^x}$$

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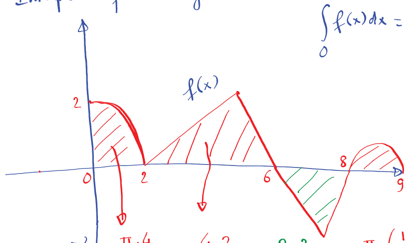
Interpret definite integrals as areas $\int_0^9 f(x)\,dx =$



$$\frac{\pi \cdot 4}{4} + \frac{4 \cdot 2}{2} - \frac{2 \cdot 3}{2} + \frac{\pi \cdot \left(\frac{1}{2}\right)^2}{2}$$

$$\pi + 4 - 3 + \frac{\pi}{8} = \boxed{\frac{9\pi}{8} + 1}$$

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HW 5.3-20

$$\int_0^5 \left|\boxed{x^2 - 1}\right| dx \stackrel{?}{=}$$

$x^2 - 1 = 0$; $(x+1)(x-1) = 0$; $x = 1$; $x = -1$

x	-1	0	1	5
x^2-1	$+$ 0	$-$	0	$+$

$$\int_0^5 |x^2-1|\,dx = \int_0^1 -(x^2-1)\,dx + \int_1^5 (x^2-1)\,dx$$

$$= \int_0^1 (1-x^2)\,dx + \int_1^5 (x^2-1)\,dx = \left(x - \frac{x^3}{3}\right)\Big|_0^1$$

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