$\left(x-\frac{x^3}{3}\right)\Big|_0^1+\left(\frac{x^3}{3}-x\right)\Big|_1^5$

$=\left(1-\frac{1}{3}\right)-0+\left(\frac{125}{3}-5\right)-\left(\frac{1}{3}-1\right)$

$=\frac{110}{3}.$

E.g. given acceleration, find position function.

A particle moves in a straight line and has acceleration given $\boxed{a(t)=6t+4}$. Its initial velocity $v(0)=-6$ cm/s. its initial position is $s(0)=9$ cm. Find the position function $s(t)$.

Nov 11-12:45 PM

Given $a(t)=6t+4$

$v(0)=-6$

$s(0)=9$

Q: $s(t)=?$

$v(t)=3t^2+4t+C_1$

$-6=v(0)=C_1$

$a(t)=v'(t)$

$v(t)=\int(6t+4)\,dt=3t^2+4t+\boxed{C_1}=-6$

$v(t)=s'(t)$

Therefore, $s(t)=\int(3t^2+4t-6)\,dt$

$s(t)=t^3+2t^2-6t+\boxed{C_2}=9$

$\boxed{s(t)=t^3+2t^2-6t+9}$

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$\int_1^2\frac{4+x^2}{x^3}\,dx=\int_1^2\left(\frac{4}{x^3}+\frac{x^2}{x^3}\right)dx=\int_1^2\left(4\cdot x^{-3}+\frac{1}{x}\right)dx$

$=\left(4\cdot\frac{x^{-2}}{-2}+\ln|x|\right)\Big|_1^2$

$=\left(\frac{-2}{x^2}+\ln|x|\right)\Big|_1^2=\left(\frac{-2}{4}+\ln 2\right)-(-2)$

$=-\frac{1}{2}+\ln 2+2=\boxed{\frac{3}{2}+\ln 2}.$

FTC, Part 2

$\int_0^4(4-t)\sqrt{t}\,dt=\int_0^4\left(4\sqrt{t}-t\sqrt{t}\right)dt=\int_0^4\left(4t^{\frac{1}{2}}-t^{\frac{3}{2}}\right)dt$

$\left(\frac{8t^{\frac{3}{2}}}{3}-\frac{2t^{\frac{5}{2}}}{5}\right)\Big|_0^4$

$\frac{64}{3}-\frac{64}{5}=..$

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Net Change Theorem.

Write an integral that quantifies the increase in the volume of a sphere as its radius double from R to $2R$.

Evaluate the integral.

$V(x)=\frac{4}{3}\pi x^3$

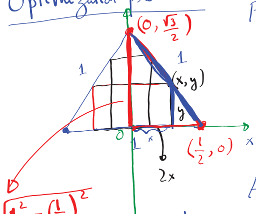
Net change (increase) $=\int_R^{2R}V'(x)\,dx=\int_R^{2R}4\pi x^2\,dx$

$=4\pi\cdot\frac{x^3}{3}\Big|_R^{2R}=4\pi\left(\frac{8R^3}{3}-\frac{R^3}{3}\right)$

$=4\pi\cdot\frac{7R^3}{3}.$

Nov 11-1:05 PM

Optimization problem.



$\sqrt{1^2-\left(\frac{1}{2}\right)^2}$

$=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$

Find the dimensions of the rectangle with the largest area that can be inscribed in an equilateral triangle of side length 1

Area $=2xy$

Line passes through $\left(0,\frac{\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2},0\right)$

Slope $=\frac{0-\frac{\sqrt{3}}{2}}{\frac{1}{2}-0}=\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\boxed{-\sqrt{3}}$

$\boxed{y=-\sqrt{3}x+\frac{\sqrt{3}}{2}}$

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$\boxed{A(x)=2x\left(-\sqrt{3}x+\frac{\sqrt{3}}{2}\right)}$

Maximize $A(x)$; $\boxed{-\frac{1}{2}\le x\le\frac{1}{2}}$

$A(x)=-2\sqrt{3}x^2+\sqrt{3}x$

$A'(x)=-4\sqrt{3}x+\sqrt{3}=0$

$x=\frac{\sqrt{3}}{4\sqrt{3}}=\boxed{\frac{1}{4}}$ ← critical pt.

$\boxed{A\left(\frac{1}{4}\right)=\frac{1}{2}\cdot\left(-\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{2}\right)}$ $A\left(-\frac{1}{2}\right)=-\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)$

$A\left(\frac{1}{2}\right)=\left(-\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)=0$

Max area occurs when $x=\frac{1}{4}$; $y=-\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{4}.$

Nov 11-1:21 PM