FTC, part 1

$\frac{d}{dx}\left(\int_{1-2x}^{1+2x} t\sin t\,dt\right)$

$\int_{7}^{1+2x} t\sin t\,dt + \int_{1-2x}^{7} t\sin t\,dt$

$\frac{d}{dx}\left(\int_{7}^{1+2x} t\sin t\,dt - \int_{7}^{1-2x} t\sin t\,dt\right)$

FTC 1: $\frac{d}{dx}\left(\int_a^{u(x)} f(t)\,dt\right) = f(u(x))\cdot u'(x)$

FTC 2: $\int_a^b f(x)\,dx = F(b) - F(a)$

$(1+2x)\sin(1+2x)\cdot 2 - (1-2x)\sin(1-2x)\cdot(-2)$

$2(1+2x)\sin(1+2x) + 2(1-2x)\sin(1-2x)$

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Average value of f over $[a,b] = \frac{1}{b-a}\int_a^b f(x)\,dx$

Ex: $v(t) = t^2 - t - 6$. ① Find displacement. $0 \le t \le 5$

② Find the distance travel $0 \le t \le 5$.

Displacement: $\int_0^5 (t^2 - t + 6)\,dt = \left(\frac{t^3}{3} - \frac{t^2}{2} - 6t\right)\Big|_0^5$

$= \frac{125}{3} - \frac{25}{2} - 30$

Distance $= \int_0^5 |t^2 - t - 6|\,dt$

$t^2 - t - 6 = 0$; $(t-3)(t+2) = 0$

$t = 3$; $t = -2$

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t: -2, 0, 3, 5

$\int_0^5 |t^2 - t - 6|\,dt = \int_0^3 -(t^2 - t - 6)\,dt + \int_3^5 (t^2 - t - 6)\,dt$

$= \left(-\frac{t^3}{3} + \frac{t^2}{2} + 6t\right)\Big|_0^3 + \left(\frac{t^3}{3} - \frac{t^2}{2} - 6t\right)\Big|_3^5$

$= \cdots$

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Right & left Riemann Sums

$f(x) = x^2 - 2x + 1$ on $[0,2]$

R_4: 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2

$\Delta x = \frac{2}{4} = \frac{1}{2}$

$R_4 = \left(f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2)\right)\cdot\frac{1}{2}$

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