

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}; \quad f(x) = mx + b \\
 &= \lim_{h \rightarrow 0} \frac{m(x+h) + b - (mx + b)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{mx + mh + b - mx - b}{h} \\
 &= \lim_{h \rightarrow 0} \frac{mh}{h} = \lim_{h \rightarrow 0} (m) = \boxed{m}.
 \end{aligned}$$

23(b) $f(x) = |x|$. Use the limit definition of the derivative to show that the derivative DNE at $x = 0$.

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$$\begin{aligned}
 f'(0) &= \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}; \quad f(x) = |x| \\
 f'(0) &= \lim_{h \rightarrow 0} \frac{|h|}{h} \quad \text{DNE} \\
 \text{Left limit: } \lim_{h \rightarrow 0^-} \frac{|h|}{h} &= \lim_{h \rightarrow 0^-} \frac{-h}{h} = \lim_{h \rightarrow 0^-} (-1) = \boxed{-1} \\
 \text{Right limit: } \lim_{h \rightarrow 0^+} \frac{|h|}{h} &= \lim_{h \rightarrow 0^+} \frac{h}{h} = \lim_{h \rightarrow 0^+} (1) = \boxed{1} \neq -1
 \end{aligned}$$

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24(a) Find the values of x for which the tangent line to the graph of $f(x) = x\sqrt{x}$ is parallel to the line $y = \frac{3}{2}x + 1$.

$$\begin{aligned}
 f(x) &= x \cdot x^{\frac{1}{2}} = x^{\frac{3}{2}} \\
 f'(x) &= \frac{3}{2} x^{\frac{1}{2}} = 3 \\
 \frac{3}{2} \sqrt{x} &= 3 \\
 \frac{\sqrt{x}}{2} &= 1; \quad \sqrt{x} = 2; \quad \boxed{x = 4}.
 \end{aligned}$$

24(b) $f(x) = ax^2 + bx + c$. Find a, b, c such that $f(2) = 5$; $f'(2) = 3$; $f''(2) = 2$.

$$\begin{aligned}
 f'(x) &= 2ax + b; \quad f''(x) = 2a; \quad f''(2) = 2a = 2; \quad \boxed{a = 1}
 \end{aligned}$$

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$$\begin{aligned}
 f'(2) &= 4a + b = 3 \\
 4 \cdot 1 + b &= 3; \quad b = -1 \\
 f(2) &= 4a + 2b + c = 5 \\
 4 + (-2) + c &= 5 \\
 2 + c &= 5; \quad c = 3 \\
 f(x) &= \boxed{x^2 - x + 3}
 \end{aligned}$$

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