HW 3.6.7

$y = \left(\boxed{f(u)+5x}\right)^{2}$; $\boxed{u = x^3 - 2x}$; $\frac{du}{dx} = 3x^2-2$

$f(4) = 8$; $\boxed{\frac{dy}{dx} = 16 \text{ when } x = 2.}$

Find $f'(4)$.

$\frac{dy}{dx} = 2\cdot(f(u)+5x)\cdot\left(\frac{d}{dx}(f(u))+5\right)$

$= 2\cdot(f(u)+5x)\cdot\left(f'(u)\cdot\frac{du}{dx}+5\right)$

$= \boxed{2(f(u)+5x)\cdot(f'(u)\cdot(3x^2-2)+5)}$

$u = x^3-2x$; when $x=2$; $u=4$

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$\frac{dy}{dx} = 2(f(u)+5x)^2\cdot(f'(u)\cdot(3x^2-2)+5)$

$x=2$

$\frac{dy}{dx}\Big|_{x=2} = \underbrace{2\cdot(f(4)+10)^2\cdot(f'(2)\cdot 10+5)}_{16}$

$2\cdot(f(4)+10)^2\cdot(f'(2)\cdot 10+5) = 16$

$2\cdot 18^2\cdot(10\cdot f'(2)+5) = 16$

$f'(2) = \left(\frac{16}{2\cdot 18^2}-5\right)\cdot\frac{1}{10}$.

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Exam 1 - Written part.

21 (a) $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{x^2+x}\right)$

Common denominator: $x(x+1)$

$= \lim_{x\to 0}\left(\frac{1\cdot(x+1)}{x(x+1)} - \frac{1}{x(x+1)}\right)$

$= \lim_{x\to 0}\left(\frac{x+1}{x(x+1)} - \frac{1}{x(x+1)}\right)$

$= \lim_{x\to 0}\frac{x+\cancel{1}-\cancel{1}}{x(x+1)}$

$= \lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}(x+1)} = \lim_{\boxed{x\to 0}}\frac{1}{x+1} = \frac{1}{1} = \boxed{1}$

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21 (b) $\lim_{x\to -4}\frac{\boxed{\sqrt{x^2+9}-5}}{x+4}$ $\left(\frac{0}{0}\right)$

$\lim_{x\to -4}\frac{(\boxed{\sqrt{x^2+9}}-\boxed{5})}{x+4}\cdot\frac{(\sqrt{x^2+9}+5)}{\sqrt{x^2+9}+5}$

$\lim_{x\to -4}\frac{x^2+9-25}{(x+4)(\sqrt{x^2+9}+5)} = \lim_{x\to -4}\frac{x^2-16}{(x+4)(\sqrt{x^2+9}+5)}$

$= \lim_{x\to -4}\frac{(x-4)\cancel{(x+4)}}{\cancel{(x+4)}(\sqrt{x^2+9}+5)} = \lim_{x\to -4}\frac{x-4}{\sqrt{x^2+9}+5}$

$= \frac{-8}{5+5} = \frac{-8}{10} = \boxed{\frac{-4}{5}}$.

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22 (a) $h(x) = \frac{\boxed{g(x)}}{\boxed{1+f(x)}}$. $\xrightarrow{\text{derivative}}$ $0 + f'(x)$

$f(2) = -3$; $g(2) = 4$; $f'(2) = -2$; $g'(2) = 7$.

Find $h'(2)$.

$h'(x) = \frac{g'(x)\cdot(1+f(x)) - g(x)\cdot f'(x)}{(1+f(x))^2}$

$h'(2) = \frac{g'(2)\cdot(1+f(2)) - g(2)\cdot f'(2)}{(1+f(2))^2}$

$= \frac{7\cdot(1-3) - 4\cdot(-2)}{(1-3)^2} = \frac{-14+8}{4} = -\frac{6}{4} = \boxed{\frac{-3}{2}}$

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22 (b) $f(2) = 10$; $\boxed{f'(x) = \boxed{x^2}\boxed{f(x)}}$ for all x.

Find $f''(2)$

$f'(2) = 4\cdot f(2)$
$= 40$

$f''(x) = 2x\cdot f(x) + x^2\cdot f'(x)$

$f''(2) = 4\boxed{f(2)} + 4\cdot\boxed{f'(2)}$

$= 4\cdot 10 + 4\cdot 40 = 40 + 160 = \boxed{200}$.

23 (c) Find the derivative $f'(x)$ of the function $\boxed{f(x) = mx+b}$ using the limit definition.

$\boxed{f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$

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