E.g. ① $\displaystyle\int \frac{2x^3+3x}{\boxed{x^4+3x^2}}\,dx$

let $u = x^4+3x^2$. Then $du = (4x^3+6x)\,dx$ du

$$\frac{1}{2}\int \frac{2\cdot(2x^3+3x)}{x^4+3x^2}\,dx = \frac{1}{2}\int \frac{\boxed{4x^3+6x}\,dx}{\boxed{x^4+3x^2}}$$

u

$$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|x^4+3x^2| + C}$$

② $\displaystyle\int_0^{\pi/2} \frac{\sin x}{\boxed{1+\cos x}}\,dx$ let $u = 1+\cos x$

Then $du = -\sin x\,dx$

$$-\int \frac{\boxed{-\sin x\,dx}^{du}}{\boxed{1+\cos x}_{u}} = -\int \frac{du}{u} = -\ln|u| + C$$

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$$= -\ln|1+\cos x|\Big|_0^{\pi/2}$$

$$= -\ln\left|1+\cos\tfrac{\pi}{2}\right| - \left(-\ln|1+\cos 0|\right)$$

$$= -\ln 1 + \ln(2) = \boxed{\ln 2}.$$

③ $\displaystyle\int \tan\theta\,d\theta = \int \frac{\sin\theta}{\cos\theta}\,d\theta = -\int \frac{\boxed{-\sin\theta\,d\theta}^{du}}{\cos\theta_{u}}$

let $u = \cos\theta$. $du = -\sin\theta\,d\theta$

$$= -\int \frac{du}{u} = -\ln|u| + C = \boxed{-\ln|\cos\theta| + C}$$

$$= \ln|(\cos\theta)^{-1}| + C = \boxed{\ln|\sec\theta| + C}$$

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④ $\displaystyle\int \frac{\ln x}{x^2}\,dx$ $\left(\int \ln x = x\ln x - x + C.\right)$

let $u = \frac{1}{x}$ Then $du = -\frac{1}{x^2}\,dx$

$$-\int \ln x \boxed{\left(-\frac{1}{x^2}\,dx\right)}_{du} = -\int \ln\left(\frac{1}{u}\right) du$$

Since $u = \frac{1}{x}$; $x = \frac{1}{u}$

$$= -\int \ln\left(u^{\boxed{-1}}\right) du$$

$$= -\int -\ln(u)\,du$$

$$= \int \ln(u)\,du = u\ln u - u + C$$

$$= \boxed{\frac{1}{x}\ln\frac{1}{x} - \frac{1}{x} + C}$$

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