## Fundamental Theorem of Calculus (Section 5.3)

F.T.C.

F.T.C., part II

If $f$ is continuous on the interval $[a,b]$ and $F(x)$ is an antiderivative of $f(x)$ over $[a,b]$, then

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

Useful notation: $F\Big|_a^b = F(b) - F(a)$

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E.g. ① $\int_0^4 x^2\,dx = \frac{x^3}{3}\Big|_0^4 = \frac{(4)^3}{3} - \frac{(0)^3}{3} = \boxed{\frac{64}{3}}$

② $\int_1^4 \frac{4}{x^2}\,dx = 4\cdot\int_1^4 \frac{1}{x^2}\,dx = 4\cdot\int_1^4 x^{-2}\,dx = 4\cdot\frac{x^{-1}}{-1}\Big|_1^4$

$= -4\cdot\frac{1}{x}\Big|_1^4$

$= -4\cdot\left(\frac{1}{4} - \frac{1}{1}\right)$

$= -4\cdot\left(-\frac{3}{4}\right) = \boxed{3}$

(Recall: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$)

③ $\int_4^8 \left(4t^{\frac{5}{2}} - 3t^{\frac{3}{2}}\right)dt$

$= \left(4\cdot\frac{t^{\frac{5}{2}+1}}{\frac{5}{2}+1} - 3\cdot\frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right)\Bigg|_4^8 = \left(4\cdot\frac{t^{\frac{7}{2}}}{\frac{7}{2}} - 3\cdot\frac{t^{\frac{5}{2}}}{\frac{5}{2}}\right)\Bigg|_4^8$

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$= \left(\frac{8}{7}t^{\frac{7}{2}} - \frac{6}{5}t^{\frac{5}{2}}\right)\Bigg|_4^8$

$= \left[\frac{8}{7}\cdot(8)^{\frac{7}{2}} - \frac{6}{5}\cdot(8)^{\frac{5}{2}}\right] - \left[\frac{8}{7}\cdot(4)^{\frac{7}{2}} - \frac{6}{5}\cdot(4)^{\frac{5}{2}}\right]$

$= \left[\frac{8}{7}\cdot(\sqrt{8})^7 - \frac{6}{5}\cdot(\sqrt{8})^5\right] - \left[\frac{8}{7}\cdot 2^7 - \frac{6}{5}\cdot 2^5\right] = \dots$

④ $\int_1^4 \frac{2-\sqrt{z}}{z^2}\,dz = \int_1^4 \left(\frac{2}{z^2} - \frac{\sqrt{z}}{z^2}\right)dz = \int_1^4 \left(2\cdot z^{-2} - z^{-\frac{3}{2}}\right)dz$

$z^{\frac{1}{2}}$

$= \left(2\cdot\frac{z^{-1}}{-1} - \frac{z^{-\frac{1}{2}}}{-\frac{1}{2}}\right)\Bigg|_1^4 = \left(\frac{-2}{z} + \frac{2}{\sqrt{z}}\right)\Bigg|_1^4$

$= \left(-\frac{2}{4} + \frac{2}{\sqrt{4}}\right) - \left(\frac{-2}{1} + \frac{2}{\sqrt{1}}\right) = \left(-\frac{1}{2} + 1\right) - (-2+2)$

$= \boxed{\frac{1}{2}}$

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⑤ $\int_0^{\pi/2} \sin(\theta)\,d\theta = -\cos(\theta)\Big|_0^{\pi/2} = \underbrace{-\cos\left(\frac{\pi}{2}\right)}_{0} - \left(-\underbrace{\cos(0)}_{1}\right)$

$= \boxed{1}$

⑥ $\int_{\pi/4}^{\pi/2} \csc^2\theta\,d\theta = -\cot\theta\Big|_{\pi/4}^{\pi/2} = \underbrace{-\cot\left(\frac{\pi}{2}\right)}_{0} - \left(-\underbrace{\cot\left(\frac{\pi}{4}\right)}_{1}\right)$

$= \boxed{1}$

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F.T.C., part 1

Suppose $f(x)$ is a continuous function on the interval $[a,b]$.

Define the function $F(x)$ by:

$$F(x) = \int_{\boxed{a}}^{\boxed{x}} f(t)\,dt$$

Then F.T.C part 1 says:

$$F'(x) = f(x) \text{ for every } x \text{ in } [a,b].$$

In short,

$$\boxed{\frac{d}{dx}}\left(\int_a^x \boxed{f(t)}\,dt\right) = f(x)$$

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E.g. $f(t) = t^2$, on $[1,3]$

Define $F(x) = \int_1^x f(t)\,dt$ for every $x$ in $[1,3]$

$F(x) = \int_1^x t^2\,dt = \frac{t^3}{3}\Big|_1^x = \frac{x^3}{3} - \frac{1}{3}.$

$F(x) = \frac{x^3}{3} - \frac{1}{3}.$

$F'(x) = x^2 = f(x)$

$\frac{d}{dx}\left(\int_1^x t^2\,dt\right) = x^2.$

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