2

$$\frac{d}{dx}\left(\int_2^x \boxed{e^{-t^2}}\,dt\right) \underset{=}{\text{FTC I}} e^{-x^2}$$

$\underbrace{\qquad}$ define a function in x, very hard to find explicit formula

$$\frac{d}{dx}\left(\int_4^{x} \frac{ds}{\sqrt{16-s^2}}\right) \underset{=}{\text{FTC 1}} \frac{1}{\sqrt{16-x^2}}$$

function of x

E.g. $\frac{d}{dx}\left(\int_1^{\boxed{x^4}} \sec(t)\,dt\right)$ this requires the Chain Rule.

let $u = x^4$. $\frac{d}{dx}\left(\boxed{\int_1^u \sec(t)\,dt}\right)$

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$\int_1^u \sec(t)\,dt$ defines a function in u

$$F(u) = \int_1^u \sec(t)\,dt.$$

$F \xrightarrow{\frac{dF}{du}} u \xrightarrow{\frac{du}{dx}} x$

$$\frac{dF}{dx} = \frac{dF}{du}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(F(u)\right) = \frac{d}{du}\left(F(u)\right)\cdot\frac{du}{dx} \qquad (u = x^4)$$

$$= \underbrace{\frac{d}{du}\left(\int_1^u \sec(t)\,dt\right)}_{\text{FTC. I}}\cdot 4x^3$$

$$= \sec(u)\cdot 4x^3 = \sec(x^4)\cdot 4x^3$$

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$$\frac{d}{dx}\left(\int_1^{\boxed{x^4}} \sec(t)\,dt\right) = \sec(x^4)\cdot(4x^3)$$

In general,

$$\frac{d}{dx}\left(\int_a^{g(x)} f(t)\,dt\right) = f(g(x))\cdot g'(x)$$

FTC - I

E.g. $\frac{d}{dx}\left(\int_{\boxed{0}}^{\sin x} \sqrt{1-t^2}\,dt\right) \underset{\text{FTC I}}{=} \sqrt{1-(\sin x)^2}\cdot\cos x$

$$= \left(\sqrt{1-\sin^2 x}\right)\cdot\cos x = \sqrt{\cos^2 x}\cdot\cos x = |\cos x|\cdot\cos x$$

(Recall: $\sin^2 x + \cos^2 x = 1$)

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E.g. $\frac{d}{dx}\left(\int_{\boxed{\sqrt{x}}}^{\boxed{2x}} (\arctan t)\,dt\right)$

$$\int_{\sqrt{x}}^{2x} \arctan(t)\,dt = \int_{\boxed{\sqrt{x}}}^{\boxed{0}} \arctan(t)\,dt + \int_{\boxed{0}}^{\boxed{2x}} \arctan(t)\,dt$$

(insert any constant we like, we property $\int_a^b f(t)\,dt = \int_a^c f(t)\,dt + \int_c^b f(t)\,dt$)

$$= \underbrace{-\int_0^{\sqrt{x}} \arctan(t)\,dt + \int_0^{2x} \arctan(t)\,dt}$$

$$\left(\int_b^a f(t)\,dt = -\int_a^b f(t)\,dt\right)$$

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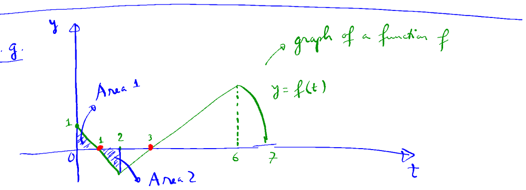
$$\frac{d}{dx}\left(\int_{\sqrt{x}}^{2x} \arctan(t)\,dt\right) = \frac{d}{dx}\left(-\int_0^{\boxed{\sqrt{x}}} \arctan(t)\,dt + \int_0^{\boxed{2x}} \arctan(t)\,dt\right)$$

$$= -\left(\arctan(\sqrt{x})\right)\cdot\frac{1}{2\sqrt{x}} + \left(\arctan(2x)\right)\cdot 2$$

$$= -\frac{\arctan(\sqrt{x})}{2\sqrt{x}} + 2\arctan(2x)$$

E.g.



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$$F(x) = \int_0^x f(t)\,dt\ ; \quad 0 \le x \le 7.$$

(1) Find $F(x)$ for $x = 0, 1, 2, 3, 4, 5, 6$.

$$F(0) = \int_0^0 f(t)\,dt = 0. \qquad F(0) = 0$$

$$F(1) = \int_0^1 f(t)\,dt = \text{Area under the graph of } f \text{ from } 0 \text{ to } 1 = \frac{1}{2}.$$

$$F(2) = \int_0^2 f(t)\,dt = \text{Area 1} - \text{Area 2} = \frac{1}{2} - \frac{1}{2} = 0$$

Similarly, find F at $3, 4, 5, 6$ by using areas.

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