u-substitution for definite integrals

E.g. $\frac{1}{6}\int_0^1 6x^2 \left(\underbrace{1+2x^3}_{u}\right)^5 dx$

let $u = 1+2x^3$. $du = 6x^2\,dx$

$\frac{1}{6}\int u^5\,du$

1st way to proceed: $\frac{1}{6}\cdot\frac{u^6}{6} = \frac{1}{36}\left(1+2x^3\right)^6\Big|_0^1$

$= \frac{1}{36}\cdot\left[\left(1+2\cdot1^3\right)^6 - \left(1+2\cdot0^3\right)^6\right]$

$= \frac{1}{36}\cdot\left[3^6-1\right] = \frac{1}{36}\cdot 728 = \ldots$

Nov 21-4:27 PM

2nd way to proceed:

to figure out the new bounds for u.

$u = 1+2x^3$.

When $x=0$; $u=1$. When $x=1$; $u=3$

$\frac{1}{6}\int_1^3 u^5\,du = \frac{1}{6}\cdot\frac{u^6}{6}\Big|_1^3 = \frac{1}{36}\cdot\left(3^6-1^6\right) = \frac{728}{36} = \ldots$

simplify.

Nov 21-4:32 PM

E.g. $\frac{1}{8}\int_0^1 8x\, e^{\overbrace{4x^2+3}^{u}}\,dx$

let $u = 4x^2+3$. $du = 8x\,dx$

$\frac{1}{8}\int e^u\,du$.

New bounds for u: when $x=0$; $u=3$;

when $x=1$; $u=7$

$\frac{1}{8}\int_3^7 e^u\,du = \frac{1}{8}\cdot e^u\Big|_3^7 = \boxed{\frac{1}{8}\left(e^7-e^3\right)}$.

Nov 21-4:34 PM

$\frac{2}{3\pi}\int_0^1 \frac{3\pi}{2}x^2 \cos\left(\underbrace{\frac{\pi}{2}x^3}_{u}\right)dx$

let $u = \frac{\pi}{2}x^3$; $du = \frac{3\pi}{2}x^2$

$\frac{2}{3\pi}\int \cos(u)\,du$.

New bounds for u: when $x=0$; $u=0$

when $x=1$; $u=\frac{\pi}{2}$.

$\frac{2}{3\pi}\int_0^{\frac{\pi}{2}} \cos(u)\,du = \frac{2}{3\pi}\cdot\sin(u)\Big|_0^{\frac{\pi}{2}} = \frac{2}{3\pi}\left(\sin\frac{\pi}{2}-\sin 0\right)$

$= \boxed{\frac{2}{3\pi}}$.

Nov 21-4:36 PM