5.7. Integrals that result in Inverse Trig functions

Basic Formulas

① $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C.$

② $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \cdot \tan^{-1}\left(\frac{x}{a}\right) + C.$

③ $\int \frac{dx}{|x|\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C$

E.g. ① $\int \frac{dx}{\sqrt{1 - 16x^2}} = \int \frac{dx}{\sqrt{1 - (4x)^2}}$ let $u = 4x$. Then $du = 4\,dx$

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$\frac{1}{4} \int \frac{\boxed{4 \cdot dx}^{\,du}}{\sqrt{1 - (\boxed{4x})^2}} = \frac{1}{4} \int \frac{du}{\sqrt{1 - u^2}} = \frac{1}{4} \sin^{-1}(u) + C$

$= \frac{1}{4} \sin^{-1}(4x) + C.$

E.g. ② $\int \frac{dx}{\sqrt{9 - x^2}} = \int \frac{dx}{\sqrt{(3)^2 - x^2}} = \sin^{-1}\left(\frac{x}{3}\right) + C$

E.g. ③ $\int \frac{dx}{25 + 4x^2} = \int \frac{dx}{25 + (2x)^2} = \frac{1}{2} \int \frac{\boxed{2dx}^{\,du}}{25 + (\boxed{2x})^2}$

let $u = 2x$. $du = 2dx$

$= \frac{1}{2} \int \frac{du}{(5)^2 + u^2} = \frac{1}{2} \cdot \frac{1}{5} \cdot \tan^{-1}\left(\frac{u}{5}\right) + C$

$= \boxed{\frac{1}{10} \cdot \tan^{-1}\left(\frac{2x}{5}\right) + C}.$

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④ $\int_0^{1/2} \frac{\sin(\boxed{\tan^{-1} t})}{1 + t^2}\, dt$ → du

let $u = \tan^{-1} t$. Then $du = \frac{1}{1 + t^2} dt$

$\int \sin(u)\, du = -\cos(u) + C = -\cos(\tan^{-1} t)\Big|_0^{\frac{1}{2}}$

$= -\cos\left(\tan^{-1}\left(\frac{1}{2}\right)\right) - \left(-\cos(\tan^{-1}(0))\right)$

$= \boxed{-\cos\left(\tan^{-1}\left(\frac{1}{2}\right)\right) + 1}$

⑤ $\int \frac{dt}{t\sqrt{1 - (\ln t)^2}}$ let $u = \ln t$. Then $du = \frac{1}{t} dt$

$= \int \frac{du}{\sqrt{1 - u^2}} = \sin^{-1}(u) + C = \boxed{\sin^{-1}(\ln t) + C}$

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