$\infty - \infty$

E.g. $\lim_{x\to 0^+}\left(\frac{1\cdot\sin x}{x\sin x} - \frac{1\cdot x}{x\sin x}\right)$

$\lim_{x\to 0^+}\frac{\sin x - x}{x\sin x} \overset{\frac{0}{0}}{=} \lim_{x\to 0^+}\frac{\cos x - 1}{\sin x + x\cos x}$

$\overset{\frac{0}{0}}{=} \lim_{x\to 0^+}\frac{-\sin x}{\cos x + \cos x - x\sin x} = \frac{0}{1+1-0} = \frac{0}{2} = \boxed{0}$

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∞^0; 1^∞; 0^0 form

Technique: similar

E.g. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$: 1^∞ form

$L = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

$\ln L = \lim_{x\to\infty}\ln\left(1+\frac{1}{x}\right)^x$

$\ln L = \lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right)$ $\quad \infty\cdot 0$

$\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$ $\quad \frac{0}{0} \to$ L'Hopital Rule

$= \lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)}$

$= \lim_{x\to\infty}\frac{1}{1+\frac{1}{x}} = \boxed{1}$

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$\ln L = 1$

$\boxed{L = e}$

$\boxed{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e}$

E.g. $\lim_{x\to 0^+} x^{\sin x}$ $\quad (0^0)$

$L = \lim_{x\to 0^+} x^{\sin x}$; $\quad \ln L = \lim_{x\to 0^+}\ln\left(x^{\sin x}\right)$

$\ln L = \lim_{x\to 0^+}(\sin x)\cdot(\ln x)$ $\quad 0\cdot\infty$

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$\lim_{x\to 0^+}\frac{\ln x}{\frac{1}{\sin x}}$ $\quad \frac{\infty}{\infty}$ $= 0$

$\lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{\cos x}{\sin^2 x}} = \lim_{x\to 0^+}\frac{1}{x}\cdot\left(-\frac{\sin^2 x}{\cos x}\right)$

$= \lim_{x\to 0^+}\frac{-\sin^2 x}{x\cdot\cos x} \overset{\frac{0}{0}}{=} \lim_{x\to 0^+}\frac{-2\sin x\cos x}{\cos x - x\sin x}$

$= \frac{0}{1-0} = \frac{0}{1} = 0$

$\ln L = 0$; $\quad \boxed{L = 1}$ $\quad \boxed{\lim_{x\to 0} x^{\sin x} = 1}$.

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