R	0		$\sqrt[3]{\frac{500}{\pi}} \approx 2.53$	
$S'(R)$		$-$	0	$+$
$S(R)$	∞	↘	local min = absolute min	↗ ∞

S is minimum when $R = \sqrt[3]{\frac{500}{\pi}} \approx 2.53$, $H = \frac{1000}{\pi \cdot (2.58)^2} \approx$ } give minimum surface area.

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E.g. 3 Find (x,y) such that the point (x,y) is closest to $(1,4)$.

Minimize Distance D

$y^2 = 2x$

Distance Formula.

(x_2, y_2), (x_1, y_1)

Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$D = \sqrt{(x-1)^2 + (y-4)^2}$

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Find (x,y) such that

$D = \sqrt{(x-1)^2 + (y-4)^2}$ is minimum

$D = \sqrt{\left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2}$

$D(y) = \sqrt{\left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2}$

Trick: Instead of minimizing $\sqrt{\ }$, we will minimize the quantity under $\sqrt{\ }$.

$Q(y) = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2$; y is in $(-\infty, \infty)$

$Q'(y) = 2\left(\frac{y^2}{2} - 1\right) \cdot y + 2(y-4)$

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$Q'(y) = 2\left(\frac{y^2}{2} - 1\right)y + 2(y-4) = 0$

$y^3 - 2y + 2y - 8 = 0$

$y^3 - 8 = 0$

$y^3 = 8$; $y = 2$ ← critical point.

y		2	
$Q'(y)$	$-$	0	$+$
$Q(y)$	↘	local min = absolute min	↗ ∞

Since $y^2 = 2x$

$4 = 2x$; $x = 2$

Conclusion: Point $(2,2)$ is the point on the parabola that is closest to $(1,4)$

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E.g. 4 3 km, 8 km

Man launches his boat from A.

Want to reach B.

Could row his boat to C and run to B

or could row his boat directly to B

could row to some point D and run to B.

row speed: 6 km/h

run speed: 8 km/h.

Q: where should he land to get to B as soon as possible.

→ Minimize the time it takes to get to B.

Find x s.t. T is minimum.

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3, 8

row speed: 6 km/h

$AD = \sqrt{x^2 + 9}$

time it takes for him to row from A to $D = \frac{\sqrt{x^2+9}}{6}$

$DB = 8 - x$

run speed: 8 km/h

time it takes for him to run from D to B is: $\frac{8-x}{8}$

Total time $T(x) = \frac{\sqrt{x^2+9}}{6} + \frac{8-x}{8}$ ← minimize

$x \geq 0$; $x \leq 8$ x belongs to the interval $[0, 8]$

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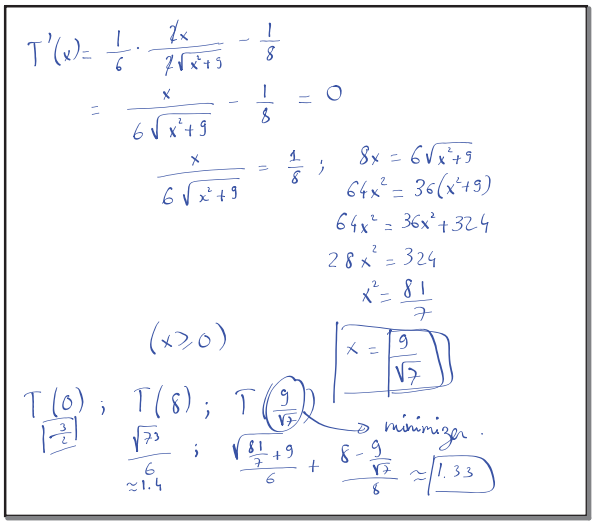
$T'(x) = \frac{1}{6} \cdot \frac{2x}{2\sqrt{x^2+9}} - \frac{1}{8}$
$= \frac{x}{6\sqrt{x^2+9}} - \frac{1}{8} = 0$
$\frac{x}{6\sqrt{x^2+9}} = \frac{1}{8}$; $8x = 6\sqrt{x^2+9}$
$64x^2 = 36(x^2+9)$
$64x^2 = 36x^2 + 324$
$28x^2 = 324$
$x^2 = \frac{81}{7}$
$(x \geq 0)$
$x = \frac{9}{\sqrt{7}}$
$T(0)$; $T(8)$; $T\left(\frac{9}{\sqrt{7}}\right)$
$\frac{3}{2}$
$\frac{\sqrt{73}}{6} \approx 1.4$;
$\frac{\sqrt{\frac{81}{7}+9}}{6} + \frac{8 - \frac{9}{\sqrt{7}}}{8} \approx 1.33$
minimizer.

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