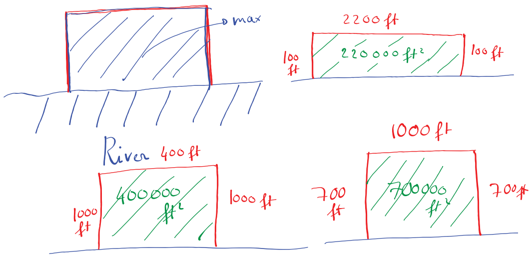
4.7. Optimization Problems

Problems in which we need to maximize or minimize a quantity.

E.g. Farmer has $\boxed{2400 \text{ ft}}$ of fencing



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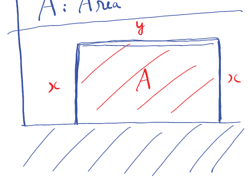
Fence off a rectangular field with the largest possible area.
(Find dimensions of the field so that the area is maximum)

Want: find x, y such that A is maximum

A: Area



$\boxed{A = xy}$

$2x + y = 2400$ ft

$\boxed{y = 2400 - 2x}$

$A = x(2400 - 2x)$

$\boxed{A(x) = 2400x - 2x^2}$ ← maximize

$x \geq 0$
$x \leq 2400$

x belongs to $\boxed{[0, 2400]}$

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$A'(x) = 2400 - 4x$

$2400 - 4x = 0$

$4x = 2400$; $\boxed{x = 600}$ ← critical point

$A(600) = 2400 \cdot 600 - 2\cdot(600)^2$

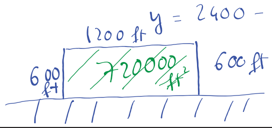
$A(600) = \boxed{720000}$

$A(0) = 2400 \cdot 0 - 2\cdot(0)^2 = 0$

$A(2400) = 2400 \cdot 2400 - 2\cdot(2400)^2 = -576000$

A is maximum when $\boxed{x = 600}$. Max Value $= 720000$

$y = 2400 - 2 \cdot 600 = 1200$



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Strategy for Solving Optimization Problems

1. Understand the problem < given / want
2. Draw a diagram.
3. Introduce some notations < Quantity to maximize / minimize Q; Unknowns (quantities that are related to Q)
4. Find an equation to relate Q to the unknowns
5. Turn Q into a function of a single variable.
6. Maximize / Minimize Q < closed interval method; 1st derivative test.

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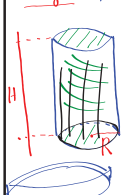
E.g. Manufacture Cylindrical Cans

(hold 1 L of oil $= \boxed{1000 \text{ cm}^3}$)

Find dimensions of the can that will minimize the cost of metal used.



minimize $\boxed{\text{Surface area of can}} = S$

Find radius R, height H such that S is minimum.

$2\pi R^2 + 2\pi R H$

$S = 2\pi R^2 + 2\pi R H$

$V = 1000 = (\text{Base area})\cdot(\text{height})$

$1000 = \pi R^2 \cdot H$; $\boxed{H = \dfrac{1000}{\pi R^2}}$

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$S = 2\pi R^2 + 2\cancel{\pi}\cancel{R} \cdot \dfrac{1000}{\cancel{\pi} R^{\cancel{2}}}$

$S = 2\pi R^2 + \dfrac{2000}{R}$; $\boxed{\underbrace{S(R) = 2\pi R^2 + \dfrac{2000}{R}}_{\text{minimize}}}$

$\boxed{R > 0}$

$S'(R) = 4\pi R - \dfrac{2000}{R^2}$

$4\pi R - \dfrac{2000}{R^2} = 0$

$4\pi R = \dfrac{2000}{R^2}$; $4\pi R^3 = 2000$

$R^3 = \dfrac{2000}{4\pi} = \dfrac{500}{\pi}$

$\boxed{R = \sqrt[3]{\dfrac{500}{\pi}}}$ ← critical point

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