4.4 Rolle's Theorem and the Mean Value Theorem

Existence Theorems

Rolle's Theorem

f : function on the interval $[a,b]$

f is continuous on $[a,b]$

f is differentiable on (a,b)

$f(a) = f(b)$

} hypotheses of Theorem

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Conclusion of Theorem:

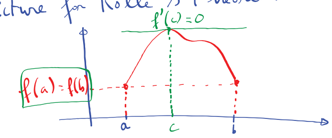
There exists a number c in (a,b) such that

$$f'(c) = 0$$

E.g. Throw object. position function — Initial position $= 0$, Final position $= 0$

velocity $= 0$ at some point.

Picture for Rolle's Theorem



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E.g. $f(x) = x^3 - 4x$. over $[-2,2]$.

(1) Verify that f satisfies all the conditions of Rolle's Theorem.

(2) Find all values of c such that $f'(c) = 0$

- continuous on $[a,b]$
- differentiable on (a,b)
- $f(a) = f(b)$

* $f(x) = x^3 - 4x$ is continuous on $[-2,2]$
* $f'(x) = 3x^2 - 4$. f is differentiable on $(-2,2)$
* $f(-2) = (-2)^3 - 4\cdot(-2) = -8 + 8 = 0$;

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$f(2) = 2^3 - 4\cdot 2 = 8 - 8 = 0$

* $f(-2) = f(2)$

All conditions of Rolle's Theorem are satisfied.

(2) Find all values of c for which $f'(c) = 0$

$f'(x) = 3x^2 - 4$

$f'(c) = 3c^2 - 4 = 0$

$3c^2 = 4$; $c^2 = \frac{4}{3}$; $c = \frac{\pm 2}{\sqrt{3}}$

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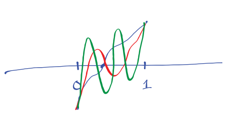
E.g. $x^3 + x - 1 = 0$

Prove that this equation has exactly one solution.

let $f(x) = x^3 + x - 1$

f is continuous.

$f(0) = -1 < 0$



$f(1) = 1 > 0$

By IVT, f has a zero in $(0,1)$

that is, the equation has a solution.

* We will use Rolle's Theorem to show that it has exactly 1 solution

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Suppose, by way of contradiction, the equation has 2 solutions a, b.

this means: $f(a) = f(b) = 0$.

Rolle's Theorem says that there exists c in (a,b) such that $f'(c) = 0$.

$f'(x) = 3x^2 + 1$

$c^2 \geq 0$

$3c^2 + 1 \geq 1$

$f'(c) = \boxed{3c^2 + 1} = \boxed{0}$ $\rightarrow \otimes$

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