Mean Value Theorem ( M.V.T).

$f$: function $[a,b]$

$f$ is continuous on $[a,b]$

$f$ is differentiable on $(a,b)$

} hypotheses of MVT

Conclusion: there exists a number $c$ in $(a,b)$ such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

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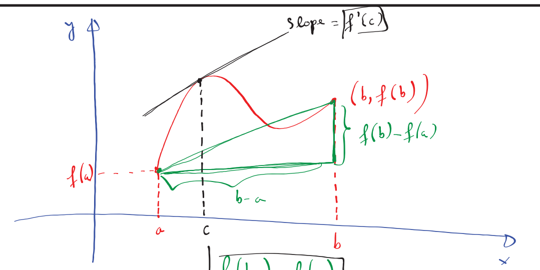


$$\boxed{\frac{f(b)-f(a)}{b-a}} = \text{Slope of secant line connecting } (a, f(a)) \text{ and } (b, f(b))$$

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E.g. T or F.

At 2pm : a car's speedometer reads 30 mi/h

At 2:10 pm : ________ 50 mi/h.

Claim : At some point between 2pm and 2:10pm, the acceleration is exactly 120 mi/h.

$f$ : velocity function.

$f(2\text{ pm}) = 30$ ; $f(2{:}10\text{pm}) = 50$ mi/h.

there is a point $c$ in $(2\text{pm}, 2{:}10\text{pm})$ such that $f'(c) = \dfrac{f(2{:}10) - f(2)}{1/6}$

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$$\boxed{f'(c)} = \frac{50-30}{\frac{1}{6}} = \frac{20}{\frac{1}{6}} = 120 \text{ mi/h.}$$

$\downarrow$ acceleration at time $c$.

E.g. $f(x) = \ln x$ , on $[1,4]$.

① Verify that $f$ satisfies all conditions of MVT.

② Find $c$ such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

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$f$ cont. on $[a,b]$

$f$ diff. on $(a,b)$

Domain of $f(x) = \ln x$ $\boxed{(0,\infty)}$

$f(x) = \ln x$ ; $\boxed{[1,4]}$

* $f$ is continuous on $[1,4]$

* $\boxed{f'(x) = \frac{1}{x}}$. $f$ is differentiable on $(1,4)$

All conditions of MVT are satisfied.

② MVT says that there exists $c$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

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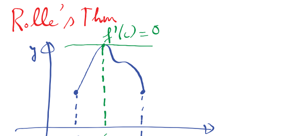
$$\frac{1}{c} = \frac{f(4)-f(1)}{4-1}$$

$$\frac{1}{c} = \frac{\ln 4 - \ln 1}{3}$$

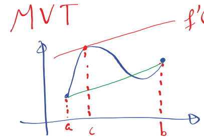
$$\frac{1}{c} = \frac{\ln 4}{3}$$

$$\boxed{c = \frac{3}{\ln 4}}.$$

Rolle's Thm




MVT $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

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