Useful Consequences of MVT

① f differentiable on an interval I.

If $f'(x) = 0$ for every x in I, then $f(x) = \text{constant}$ for every x in I.

Proof: Goal: to show $f(x) = \text{constant}$ for every x in I, we just need to pick 2 points a, b in I and show that $f(a) = f(b)$

Take $a \neq b$ in I.

f' exists on (a,b) : f is diff. on (a,b), f is cont. on $[a,b]$

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By MVT, there exists c in (a,b) such that

$$\boxed{f'(c)} = \frac{f(b) - f(a)}{b - a}.$$

$f'(c) = 0$

So, $\dfrac{f(b) - f(a)}{b - a} = 0$

$f(b) - f(a) = 0$

$\boxed{f(b) = f(a)} \Rightarrow f$ is the constant function.

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② If f and g are differentiable functions on an interval I and $f'(x) = g'(x)$ for every x in I, then $f(x) = g(x) + C$ for some constant C.

Proof: $h(x) = f(x) - g(x)$

$h'(x) = f'(x) - g'(x) = 0$

$h'(x) = 0$ for every x in I.

From ①, $h(x) = C$ for all x in I.

$f(x) - g(x) = C \Rightarrow f(x) = g(x) + C$.

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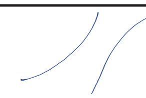
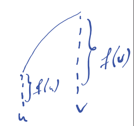
③ f cont. on $[a,b]$, diff. on (a,b)



(i) $\boxed{f'(x) > 0}$ for all x in (a,b) then f is increasing on (a,b)

(ii) $f'(x) < 0$ for all x in (a,b) then f is decreasing on (a,b)



Proof: u, v in (a,b). $u < v$.

M.V.T. applies to f on $[u,v]$

then is c such that $\underbrace{\boxed{f'(c)}}_{>0} = \dfrac{f(v) - f(u)}{v - u}$

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$\dfrac{f(v) - f(u)}{v - u} > 0$ $\quad$ $u < v$, $v - u > 0$

$f(v) - f(u) > 0$

$f(v) > f(u)$.

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