

5.6. Rational Functions

E.g. $\frac{x^2 - 2x + 3}{x^2 + 4}$

Definition of a rational function

A rational function is a function of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials.

Goals: (1) Domain of rational functions.
x-intercepts

(2) Vertical Asymptotes (V.A.) • Horizontal Asymptotes (H.A.) • Slant Asymptotes (S.A.)

(3) Graph a rational function.

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Domain of Rational Functions

E.g. $f(x) = \frac{x-1}{x+2}$

Find domain? $x+2=0$; $x=-2$

Domain: $(-\infty, -2) \cup (-2, \infty)$

$f(x) = \frac{5+2x^2}{2-x-x^2}$. Find domain?

$2-x-x^2=0$

$(x+2)(-x+1)=0$

$x+2=0$ or $-x+1=0$
 $x=-2$; $x=1$

Domain: $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

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To find domain of $f(x) = \frac{p(x)}{q(x)}$
Set $q(x)=0$. Solve for x . Domain: everything except for the values of x we just solved for.

x-intercepts

E.g. $g(x) = \frac{2x-3}{x+4}$. Find x-intercepts.

$\frac{2x-3}{x+4}=0$; $2x-3=0$ x-intercept
 $x=\frac{3}{2}$ $(\frac{3}{2}, 0)$

E.g. $w(x) = \frac{(x-1)(x+3)(x-5)}{(x+2)^2(x-4)}$. Find x-intercepts.
 $(x-1)(x+3)(x-5)=0$. $(1,0)$, $(-3,0)$, $(5,0)$
 $x=1$; -3 ; 5

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Vertical Asymptotes

Arrow Notation

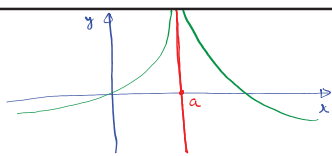
$f(x) = \frac{1}{x}$

As $x \rightarrow 0^+$, $f(x) \rightarrow \infty$
(Read: as x approaches 0 from the right, $f(x)$ approaches infinity)

As $x \rightarrow 0^-$, $f(x) \rightarrow -\infty$
(Read: as x approaches 0 from the left, $f(x)$ approaches negative infinity)

V.A.

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Definition of a vertical asymptote of a rational function.

The vertical line $x=a$ is a V.A. of $f(x)$ if as $x \rightarrow a^+$ (or $x \rightarrow a^-$), $f(x) \rightarrow \infty$ (or $f(x) \rightarrow -\infty$)

$f(x) = \frac{x(x+2)}{x(x+3)}$

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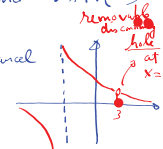
Method of finding V.A.(s) of rational functions

- (1) Factor the numerator and denominator completely
- (2) Cancel the common factors of denom. and num.
- (3) After cancellation, the values of x for which the denominator $= 0$ correspond to the V.A. of the rational function.

E.g. $f(x) = \frac{x^2 - 4x + 3}{x^2 - 9}$. Find V.A.(s)

Factor $f(x) = \frac{(x-1)(x-3)}{(x-3)(x+3)}$ (2) Cancel

(3) V.A. $x = -3$



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