Find the V. A.(s) and Removable discontinuity of the given function

(1) $u(x) = \dfrac{x^2-1}{x^2-2x-3}$

(2) $v(x) = \dfrac{3x-4}{x^3-16x}$

(1) $u(x) = \dfrac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(x-3)}$

V. A. $\boxed{x=3}$ • Removable discontinuity at $x=-1$

Hole: $\left(1, \frac{1}{2}\right)$

To find y-coordinate of hole: plug $x=-1$ into $\dfrac{x-1}{x-3}$

$\dfrac{-2}{-4} = \dfrac{1}{2}$

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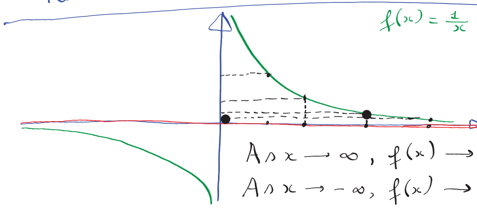
$v(x) = \dfrac{3x-4}{x^3-16x} = \dfrac{3x-4}{x(x^2-16)} = \dfrac{3x-4}{x(x-4)(x+4)}$

V.A. $\boxed{x=0;\ x=4;\ x=-4}$

Removable Discontinuity: None.



As $x \to \infty$, $f(x) \to 0$

As $x \to -\infty$, $f(x) \to 0$

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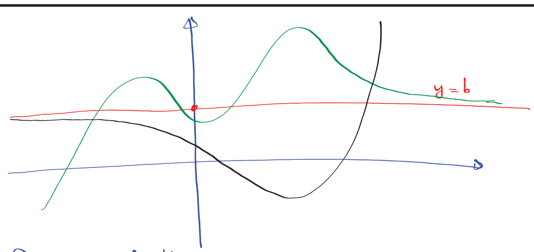

Definition of H.A.

The horizontal line $y=b$ is a H.A. of $f(x)$ if $f(x) \to b$ as $x \to \infty$ or $x \to -\infty$

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Method of finding H.A. of rational functions

$f(x) = \dfrac{p(x)}{q(x)}$

(1) Degree of $p(x)$ < Degree of $q(x)$ (E.g. $f(x) = \dfrac{x+3}{5x^2-2}$)

then $\boxed{y=0}$ is the H.A. of $f(x)$

(2) Degree of $p(x)$ = Degree of $q(x)$ (E.g. $f(x) = \dfrac{-2x^3+1}{5x^3+x^2+1}$)

then $\boxed{y = \frac{a}{b}}$ is the H.A. of $f(x)$ where a is the leading coefficient of p and b is the leading coefficient of q

(3) Degree of $p(x)$ > Degree of $q(x)$ (E.g. $f(x) = \dfrac{x^4+1}{x^2+1}$)

NO H.A.

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Why: (1) $f(x) = \dfrac{x+3}{5x^2-2} \approx \dfrac{x}{5x^2} = \dfrac{1}{5x} \approx 0$

when x is large   when x is large

$y=0$ is the H.A.

(2) $f(x) = \dfrac{-2x^3+1}{\boxed{5x^3}+x^2+1} \approx \dfrac{-2x^3}{5x^3} = \boxed{-\dfrac{2}{5}}$

$y = -\dfrac{2}{5}$ is the H.A.

(3) $f(x) = \dfrac{x^4+1}{x^2+3} \approx \dfrac{x^4}{x^2} = x^2$

NO H.A.

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Graph Rational Functions

E.g. $a(x) = \dfrac{2x-3}{x+4}$

$x = -5$

$\dfrac{2(-5)-3}{-5+4} = \dfrac{-13}{-1} = 13$

* Domain: $(-\infty, -4) \cup (-4, \infty)$
* x-intercept: $\left(\frac{3}{2}, 0\right)$
* y-intercept: $\left(0, -\frac{3}{4}\right)$
* V. A. $x = -4$
* H.A. $y = 2$

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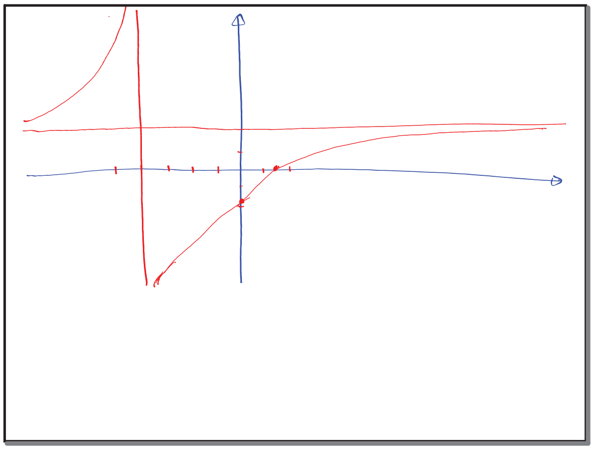

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