CALCULUS I LEARNING GOALS

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Abstract. List of learning goals for all sections of Chapter 1 through 5 and section 6.1 of the book Calculus - OpenStax Volume 1. Each learning goal is accompanied by selected practice exercises from the book.

Important Note:

- The page numbers and the section numbers here correspond to the **downloaded pdf version** of the textbook.
- For extra credit: Only turn in the exercises labeled **Assigned Written Exercises**.
- Do NOT turn in the exercises labeled **similar odd-numbered exercises**, these are for practice only, and the answers are provided online.
- Do read the suggested examples. They will help you understand the material and solve the exercises.
- At some point in this document, **Assigned Written Exercises** is abbreviated as **AWE**, **similar odd-numbered exercises** is abbreviated as **SOE**, and **Reading** is abbreviated as **R**.
- The answers to the odd-numbered exercises can be found by going to the website `https://openstax.org/details/calculus-volume-1`, click on **View Online** and go to the corresponding sections and exercises.

2. Limits

2.1. A Preview of Calculus.

(1) Understand the tangent line problem: be able to find the slopes of the secant lines passing through pairs of points on the graph of a given function and use the slopes of the secant lines to estimate the slope of the tangent line at a point.
Assigned Written Exercises (can be turned in for extra credit): **4, 5, 6 Page 134**.
Similar example from the book: Read Example 2.1 page 129.
Similar odd-numbered exercises with answers given at the website for the book (**for practice only, not to turn in**): 1-3 page 134, 13-15 page 135.

(2) Find the average velocities of an object over given time intervals and use the average velocities to estimate the instantaneous velocity at a point of time.
Assigned Written Exercises (can be turned in for extra credit): **20, 21 Page 135**.
Similar example from the book: Read Example 2.2 page 130.

(3) The area problem: estimate the area between the x-axis and the graph of a function using rectangles.
Assigned Written Exercises (can be turned in for extra credit): **28, 29 Page 136**.
Reading: Example 2.3 page 132.

2.2. The Limit of a Function.

(1) Evaluate a limit using a table of functional values.
Assigned Written Exercises: 36, 37, 38 page 156 and 157.
Reading: Example 2.4, 2.5 page 139 and 140.
Similar odd-numbered exercises: 35, 41, 43.

(2) Analyze a limit that fails to exist.
Assigned Written Exercises: 40, 44 page 157 and 158.
Reading: Example 2.7 page 143.
Similar odd-numbered exercises: 45.

(3) Use the graph of a function to find one-sided and two-sided limits.
Assigned Written Exercises: 55, 56, 57, 58 Page 159.
Reading: Example 2.6 and 2.8 pages 141, 142, 146 and 147.
(4) Understand infinite limits intuitively.
Assigned Written Exercises: 78 and 80 page 160.
Reading: Example 2.9 and 2.11 pg 148, 149, 150, 152 and 153.
Similar odd-numbered exercises: 77 and 79.

2.3. **The Limit Laws.**

(1) Evaluate a limit using limit laws.
Assigned Written Exercises: 84, 90, 92, 108, 110, 116, 120.
Reading: Example 2.14, 2.15, 2.16, 2.21, 2.22.
Similar odd-numbered exercises: 85, 89, 109, 111, 113, 117, 119, 125.
(2) Evaluate a limit in the indeterminate form 0/0.
Assigned Written Exercises: 98, 100, 102, 106.
Reading: Example 2.17, 2.18, 2.19, 2.20, 2.23.
Similar odd-numbered exercises: 93, 97, 105.
(3) Evaluate trigonometric limits.
Assigned Written Exercises: Review Exercise 216 page 214.
Reading: Example 2.25.
Similar odd-numbered exercises: 99.
(4) Evaluate limits using the Squeeze Theorem.
Assigned Written Exercises: 126 and Review Exercise 224 page 214.
Reading: Example 2.24.
Similar odd-numbered exercises: 127.

2.4. **Continuity.**

(1) Determine whether a function is continuous at a given point.
Assigned Written Exercises: 140, 144, 154.
Reading: Example 2.26, 2.27 and 2.28
Similar odd-numbered exercises: 139, 141, 143.
(2) Understand the three types of discontinuities: removable, jump and infinite discontinuity. Be able to classify the type of discontinuity of a function.
Assigned Written Exercises: 136, 138, 156.
Reading: Example 2.30, 2.31 and 2.32.
Similar odd-numbered exercises: 133, 137, 155.
(3) Understand left and right continuity and continuity over an interval and solve related problems.
Assigned Written Exercises: 146, 148, 158, 166. (Note: the given interval mentioned in these problems is $\mathbb{R}$)
Reading: Example 2.33, 2.34.
Similar odd-numbered exercises: 145, 147, 157.
(4) Know when to apply and how to apply the Intermediate Value Theorem (IVT).
Assigned Written Exercises: 150, 152, 164.
Reading: Example 2.36, 2.37, 2.38.
Similar odd-numbered exercises: 153, 165.

2.5. **The Precise Definition of a Limit.**

(1) Understand the ϵ-δ definition of a limit. Given an ϵ, be able to find the largest possible δ such that the conditional statement in the definition is satisfied.
Assigned Written Exercises: 180, 182, 184, 186.
Similar odd-numbered exercises: 181, 183, 185, 187.

3. Derivatives

3.1. **Defining the Derivative.**

(1) Find the slope and the equation of the tangent line to the graph of a function f at the point where $x = a$ using either the definition $m_{tan} = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$ or $m_{tan} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$.
AWE: 16, 20.
R: Example 3.2 and 3.3
SOE: 17, 19.

(2) Find $f'(a)$ using the definition $f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$ or $\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$.
AWE: 24.
R: Example 3.5, 3.6.
SOE: 23.

(3) Find $f'(a)$ given the graph of f
AWE: 40.
SOE: 39.

(4) Use the limit definition of the derivative to show that the derivative does not exist at a given point for a given function.
AWE: 42.
SOE: 41, 43.

(5) Find the instantaneous rate of change of a quantity at a given point.
AWE: 36.
R: Example 3.9, 3.10.
SOE: 37.

3.2. **The Derivative as a Function.**

(1) Understand the definition of the derivative as a function. Find $f'(x)$ using the definition.
AWE: 62, 68, 70, 76.
R: Example 3.11 and 3.12.
SOE: 63, 69, 73, 75.

(2) Sketch the graph of the derivative f' use the graph of the original function f.
AWE: 64, 66, 96.
R: Example 3.13.
SOE: 65, 67.

(3) Use the graph of f to find f' at given points.
AWE: 80

(4) Understand the relationship between differentiability and continuity.
R: Example 3.14.

(5) Find the second derivative using the definition.
AWE: 82.
R: Example 3.15 and 3.16.
SOE: 81, 83.

3.3. **Differentiation Rules.**

(1) Find derivatives using basic differentiation rules: constant, constant multiple, power rules, sum and difference rules, product and quotient rules.
AWE: 110, 112, 116, 126, 128, 130, 132, 144, 148.
R: Example 3.17, 3.18, 3.19, 3.20, 3.21, 3.24, 3.25, 3.26, 3.27, 3.28, 3.29, 3.30 and 3.32.
SOE: 111, 113, 117, 129, 131, 145, 147.

(2) Find the equation of the tangent line to the graph of a function. Determine the x-values at which the slope of the tangent line is satisfied some given condition.
AWE: 120, 140, 142.

R: Example 3.22 and 3.31.
SOE: 121, 139, 141.

3.4. **Derivatives as Rate of Change.**

(1) Apply the derivative to solve applications that involve rates of change: position, velocity and acceleration of moving objects, population growth, marginal cost and revenue.
AWE: 154, 158, 160, 164.
R: Example 3.34, 3.35, 3.36, 3.37, and 3.38.
SOE: 155, 159, 161, 165.

3.5. **Derivatives of Trigonometric Functions.**

(1) Find the derivatives of functions that involve basic trigonometric functions.
AWE: **180, 182, 198.**
R: Example 3.39, 3.40, 3.41, 3.44.
SOE: 181, 183, 197.
(2) Find higher-order derivatives of functions that involve basic trigonometric functions.
AWE: 196, 210.
R: 3.45, 3.46.
SOE: 211, 213.

3.6. **The Chain Rule.**

(1) Find derivatives and solve related problems using the Chain Rule.
AWE: 218, 222, 224, 228, 230, 234, 236, 238, 240, 248, 252, 254, 256.
R: Example 3.48, 3.49, 3.50, 3.51, 3.52, 3.53, 3.54, 3.55, 3.57, 3.58, 3.59.
SOE: 219, 227, 231, 237, 239, 247, 251, 253, 257.

3.7. **Derivatives of Inverse Functions.**

(1) Apply the Inverse Function Theorem to find the derivative of the inverse function of a function at a point.
AWE: 262, 272, 278, 290.
R: Example 3.61.
SOE: 261, 273, 277, 291.
(2) Find the derivatives of functions that involve inverse trigonometric functions.
AWE: 280, 284, 288, 296, 298.
R: Example 3.65, 3.66, 3.67.
SOE: 283, 285, 297, 297, 299.

3.8. **Implicit Differentiation.**

(1) Find the derivatives using implicit differentiation.
AWE: 304, 308, 322, 324.
R: Example 3.68, 3.69, 3.70, 3.73.
SOE: 305, 307, 325.
(2) Find the equation of tangent lines implicitly.
AWE: 312, 316, 318, 320.
R: Example 3.71, 3.72.
SOE: 313, 317, 319, 321.

3.9. **Derivatives of Exponential and Logarithmic Functions.**

(1) Find the derivative of an exponential function and solve application problems that involve exponential functions.
AWE: 332, 334, 338, 360, 362.
R: Example 3.74, 3.75, and 3.76.
SOE: 333, 335, 339, 359.
(2) Find the derivative of a logarithmic function and solve related problems.
AWE: 340, 344, 356.
R: Example 3.77, 3.78, 3.79, and 3.80.
SOE: 341, 343, 345.
(3) Apply the technique of logarithmic differentiation to find the derivative of a function of the form $h(x) = g(x)^{f(x)}$ or to find the derivative of complicated functions with products and quotients in an easier way.
AWE: 346, 350, 352.
R: Example 3.81, 3.82, and 3.83.
SOE: 347, 349, 353.

4. Applications of Derivatives

4.1. **Related Rates.**

(1) Solve related rates problems.
AWE: 4, 6, 10, 20, 30, 40.
R: Example 4.1, 4.2, 4.3 and 4.4.
SOE: 5, 11, 21, 31.

4.2. **Linear Approximation and Differentials.**

(1) Find the linear approximation of a function at a point and use it to approximate the value of an expression.
AWE: 54, 66.
R: Example 4.5, 4.6, and 4.7.
SOE: 55, 67.
(2) Compute the differential of a given function and approximate change with differentials.
AWE: 70, 76.
R: Example 4.8 and 4.9.
SOE: 71, 75.
(3) Using differentials to estimate errors.
AWE: 82, 84, 86.
R: Example 4.10 and 4.11.
SOE: 83, 85.

4.3. **Maxima and Minima.**

(1) Determine the critical points of a given function.
AWE: 114, 116.
R: Example 4.12.
SOE: 113, 115.
(2) Locating absolute extrema of a function over a closed interval.
AWE: 106, 122, 124, 128.
R: Example 4.13.
SOE: 105, 121, 127.

4.4. **The Mean Value Theorem.**

(1) Understand and be able to use Rolle's Theorem.
AWE: 182.
R: Example 4.14.

(2) Understand and be able to use the Mean Value Theorem and its corollaries.
AWE: 154, 164, 168, 170, 172, 188.
R: Example 4.15 and 4.16.
SOE: 155, 165, 167, 177, 189.

4.5. **Derivatives and the Shape of a Graph.**

(1) Apply the First Derivative Test to determine the intervals of increasing/decreasing and the local extrema of a given function.
AWE: 202, 206, 210, 216, 226a and 226b, 232a and 232b, 240a and 240b.
R: Example 4.17 and 4.18.
SOE: 201, 207, 217, 225a and b, 233 a and b, 239a and b.

(2) Use the second derivative to determine the intervals of concavity or local extrema of a given function.
AWE: 196, 218, 226c and 226d, 232c and 232d, 240c and 240d.
R: Example 4.19, 4.20.
SOE: 217, 225c and d, 233c and d, 239c and d.

4.6. **Limits at Infinity and Asymptotes.**

(1) Compute limits at infinity and use the result to determine the horizontal asymptote(s) and the end behavior of a function.
AWE: 264, 268, 270, 272, 274.
R: Example 4.21, 4.24, 4.25, 4.26, and 4.27.
SOE: 263, 269, 271, 273.

(2) Use the guidelines in this section to draw the graph of a given function.
AWE: 296, 300, 304.
R: Example 4.28, 4.29, 4.30 and 4.31.
SOE: 297, 299, 305.

4.7. **Applied Optimization Problems.**

(1) Set up and solve optimization problems.
AWE: 316, 320, 322, 334, 336, 344, 350, 354.
R: Example 4.32, 4.33, 4.34, 4.35, 4.36, 4.37.
SOE: 319, 333, 335, 343, 345, 349, 351.

4.8. **L'Hôpital's Rule.**

(1) Apply L'Hôpital's Rule to find indeterminate limits of the type $0/0$, ∞/∞, $0\cdot\infty$, $\infty-\infty$, ∞^0, and 0^0.
AWE: 356, 360, 362, 370, 374, 380, 382, 390, 392, 394, 402.
R: Example 4.38, 4.39, 4.40, 4.41, 4.42, 4.43, and 4.44.
SOE: 357, 359, 371, 373, 383, 391, 393, 395.

4.9. **Newton's Method.**

(1) Approximate a root of a polynomial using Newton's method.
AWE: 410, 414, 426, 434, 436, 452, 464.
R: Example 4.46 and 4.47.
SOE: 407, 415, 425, 435, 453.

(2) Recognize when Newton's method fails.
AWE: 448.
R: Example 4.48.

4.10. **Antiderivatives.**

(1) Find the general antiderivative of a given function. Verify and evaluate indefinite integrals.
AWE: 472, 476, 480, 484, 488, 490, 492, 496, 498.
R: Example 4.50, 4.51, 4.52.
SOE: 471, 477, 481, 485, 487, 493.

(2) Solve basic initial-value problems.
AWE: 502, 506, 510, 514, 520.
R: Example 4.53 and 4.54.
SOE: 501, 507, 509, 513, 519.

5. Integration

5.1. **Approximating Areas.**

(1) Understand sigma notation and be able to use sigma notation and basic summation formulas to evaluate sums.
AWE: 2, 6, 8.
R: Example 5.1, 5.2 and 5.3.
SOE: 3, 7, 9.

(2) Approximate the area under a curve using either the left-endpoint approximation or right-endpoint approximation.
AWE: 14, 16, 26.
R: Example 5.4.
SOE: 15, 17, 27.

(3) Know what a Riemann sum is. Be able to form and evaluate a Riemann sum for a given function.
AWE: 22, 44, 54, 56.
R: Example 5.5 and 5.6.
SOE: 23, 45, 53.

5.2. **The Definite Integrals.**

(1) Understand the definition of the definite integral as the limit of a Riemann sum and solve related problems.
AWE: 62, 66.
R: Example 5.7.
SOE: 63, 67.

(2) Understand the relation between area and the definite integral. Evaluate definite integrals using geometry.
AWE: 70, 72, 76, 78, 82.
R: Example 5.8, 5.9 and 5.10.
SOE: 71, 73, 75, 77, 81.

(3) Apply the properties of the definite integral to solve problems.
AWE: 92, 98, 102, 104, 108, 124, 134, 138.
R: Example 5.11, 5.12, and 5.13.
SOE: 93, 99, 105, 109, 125, 135.

(4) Find the average value of a function using the definite integral.
AWE: 112, 114, 126 (Hint: use 124), 132.
R: Example 5.14.
SOE: 113, 115, 127.

5.3. **The Fundamental Theorem of Calculus.**

(1) Apply the Fundamental Theorem of Calculus - Part I to find derivatives and solve related problems.
AWE: 148, 152, 154, 156, 160, 162.
R: Example 5.17, 5.18, and 5.19.
SOE: 149, 153, 155, 157, 161, 163.

(2) Apply the Fundamental Theorem of Calculus - Part II to evaluate definite integrals and solve related problems.
AWE: 172, 176, 180, 182, 184, 186, 188, 196.
R: Example 5.20, 5.21, and 5.22.
SOE: 173, 177, 179, 183, 185, 187, 197.

5.4. **Integration Formulas and the Net Change Theorem.**

(1) Apply the basic integration rules and the Net Change Theorem to solve application problems.
AWE: 216, 220, 224, 226, 230, 232, 238.
R: Example 5.24, 5.25, 5.26 and 5.27.
SOE: 215, 217, 223, 225, 229, 231.

(2) Understand the properties of integrals of even and odd functions.
R: Example 5.28 and 5.29.

5.5. **Substitution.**

(1) Apply the method of Integration by Substitution to find indefinite, definite integrals and the area under a curve.
AWE: 266, 270, 276, 280, 282, 284, 292, 296, 314, 316.
R: Example 5.30, 5.31, 5.32, 5.33, 5.34, 5.35, 5.36.
SOE: 265, 269, 275, 279, 283, 285, 293, 297, 315, 317.

5.6. **Integrals Involving Exponential and Logarithmic Functions.**

(1) Integrate functions involving exponential functions and solve application problems.
AWE: 322, 338, 340, 354, 364, 376.
R: Example 5.37, 5.38, 5.39, 5.40, 5.41, 5.42 5.43 and 5.44.
SOE: 323, 339, 341, 377.

(2) Integrate functions involving logarithmic functions and solve application problems.
AWE: 330, 334, 344, 350, 358, 362, 372.
R: Example 5.45, 5.46, 5.47, and 5.48.
SOE: 329, 335, 345, 349, 355, 359, 363, 373.

5.7. **Integrals Resulting in Inverse Trigonometric Functions.**

(1) Find integrals resulting in inverse trigonometric functions and solve related problems.
AWE: 392, 394, 396, 402, 406, 412, 414, 424, 428, 432, 434, 436, 438.
R: Example 5.49, 5.50, 5.51, 5.52, 5.53, 5.54.
SOE: 391, 393, 395, 401, 405, 411, 415, 423, 427, 431, 433, 437.

6. Applications of Integration

6.1. **Areas between Curves.**

(1) Find the area of a region between two curves.
AWE: 2, 4, 6, 8, 10, 16, 22, 32, 36, 48.
R: Example 6.1, 6.2, 6.3, 6.4 and 6.5.
SOE: 1, 3, 5, 7, 11, 17, 25, 37, 57.