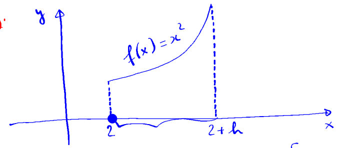
12.1 Introduction to Limits

E.g.



Average R.O.C of $f(x)=x^2$ over $[2, 2+h]$

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{f(2+h) - f(2)}{2+h-2} = \frac{(2+h)^2 - 4}{h}$$

If we think about f as a position function of a moving object, then this quantity represents the average velocity of the object.

Nov 28-6:46 PM

Q. What if we want to find the instantaneous velocity at time $x = 2$?

$\frac{(2+h)^2 - 4}{h}$ → average velocity over $[2, 2+h]$

A: Choose smaller and smaller values of h and plug them into this quantity.

$\frac{(2+h)^2-4}{h}$ approaches 4 as h approaches 0

Notation: $\lim_{h \to 0} \frac{(2+h)^2 - 4}{h} = 4.$

Nov 28-6:52 PM

In general, the notation

$$\lim_{x \to a} f(x)$$

gives us the answer to the question: where does $f(x)$ approach as x approaches the # a ?

$\lim_{x \to 0} \frac{\sin x}{x} = 1.$ (See table)

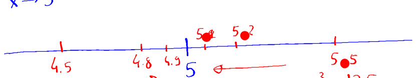
Nov 28-7:01 PM

h	average velocity over [2,2+h]	x	sin(x)/x
0.5	4.5	0.5	0.95885
0.2	4.2	0.2	0.99335
0.1	4.1	0.1	0.99833
0.01	4.01	0.01	0.99998
0.001	4.001	0.001	1
0.0001	4.0001	0.0001	1
0.00001	4.00001	0.00001	1

Nov 28-7:06 PM

More examples of finding limits numerically

$$\lim_{x \to 5} \frac{x^3 - 125}{x - 5}$$



when $x \to 5$ from the right, $\frac{x^3-125}{x-5} \to 75$

when $x \to 5$ from the left, $\frac{x^3-125}{x-5} \to 75$

Notation: $x \to 5$ from the right is denoted by $x \to 5^+$

$$\lim_{x \to 5^+} \frac{x^3-125}{x-5} = 75$$

$x \to 5$ from the left is denoted by $x \to 5^-$

$$\lim_{x \to 5^-} \frac{x^3-125}{x-5} = 75$$

Nov 28-7:15 PM

In general, for $\boxed{\lim_{x \to a} f(x)}$ to exist, both $\boxed{\lim_{x \to a^+} f(x)}$ and $\boxed{\lim_{x \to a^-} f(x)}$ must be equal to the same #.

E.g. Consider: $\lim_{x \to 7^+} \frac{|x-7|}{x-7} = 1$

$\lim_{x \to 7^-} \frac{|x-7|}{x-7} = -1$

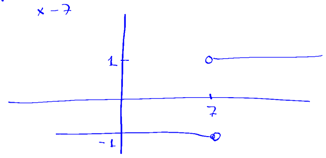
$$\boxed{\frac{|x-7|}{x-7}}$$

$$|x-7| = \begin{cases} x-7 & \text{when } x > 7 \\ -(x-7) & \text{when } x < 7 \end{cases}$$

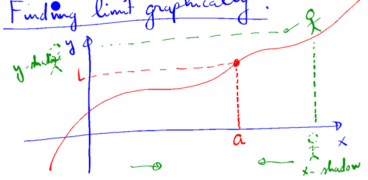
$$\frac{|x-7|}{x-7} = \begin{cases} \frac{x-7}{x-7} & \text{when } x > 7 \\ \frac{-(x-7)}{x-7} & \text{when } x < 7 \end{cases} = \begin{cases} 1 & \text{when } x > 7 \\ -1 & \text{when } x < 7 \end{cases}$$

Nov 28-7:24 PM

Graph of $\frac{|x-7|}{x-7}$ looks like



Finding limit graphically:



Nov 28-7:33 PM