3.4 Composition of functions

Goals:

1. Perform algebraic operations on 2 given functions and find the domain of the resulting function.
2. Create a function by composition of functions. Find the domain and evaluate composite functions
3. Decompose a composite function into its component functions.

Algebraic operations on functions

Sep 20-11:27 AM

E.g. $f(x) = x^3 + 10x - 3x^2 + 7$

$g(x) = x^2 + 7$.

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(fg)(x) = f(x)\,g(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

} Combine functions

$(f+g)(x) = x^3 - 2x^2 + 10x + 14$.

Sep 20-11:36 AM

$(f-g)(x) = (x^3 + 10x - 3x^2 + 7) - (x^2 + 7)$

$= x^3 + 10x - 4x^2$.

$(g-f)(x) = (x^2+7) - (x^3 + 10x - 3x^2 + 7)$

$= -x^3 - 10x + 4x^2$

$(fg)(x) = f(x)\,g(x) = (x^3 + 10x - 3x^2 + 7)(x^2 + 7)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^3 + 10x - 3x^2 + 7}{x^2 + 7}$

Domain: $(-\infty, \infty)$.

Sep 20-11:41 AM

Composing Functions:

option 1: 10% off your purchase, $10 off

option 2: $10 off, 10% off

$100 → $90 → $80 Save $1

$100 → $90 → $81

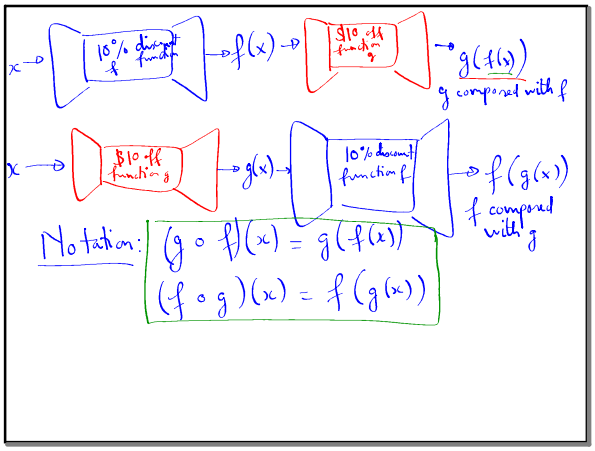
$1000 → option 1: $900 → $891

option 2: $990 → $891

option 1: $\$x$ → $x - \frac{1}{10}x = \frac{9}{10}x$ → $\frac{9}{10}x - 10$

option 2: x → $x - 10$ → $(x-10) - \frac{1}{10}(x-10) = \frac{9}{10}(x-10) = \frac{9}{10}x - 9$

Sep 20-11:46 AM



Notation: $(g \circ f)(x) = g(f(x))$

$(f \circ g)(x) = f(g(x))$

Sep 20-11:54 AM

E.g. $f(x) = 2x + 1$; $g(x) = 3 - x$

Find $f \circ g$ and $g \circ f$

$(f \circ g)(x) = f(g(x))$

$= f(3 - x)$

$= 2(3 - x) + 1$

$(f \circ g)(x) = 6 - 2x + 1 = -2x + 7$

$(g \circ f)(x) = g(f(x)) = g(2x + 1)$

$(g \circ f)(x) = 3 - (2x + 1) = -2x + 2$

Sep 20-11:59 AM

Order matters when we compose functions.
Function composition is not commutative.

$f \circ g \neq g \circ f$ in general.

Evaluate Composite functions

Evaluate Composite functions using tables

x	$f(x)$	$g(x)$
1	2	5
2	4	3
3	6	1
4	8	0

$(f \circ g)(3) = f(g(3)) = f(1) = 2$

$(g \circ g)(2) = g(g(2)) = g(3) = 1$.

Sep 20-12:04 PM

Evaluate Composite functions using graphs

Evaluate Composite functions using formulas

E.g. $f(t) = t^2 - t$. $h(u) = 3u + 2$

Find $h(f(2))$ and $h(f(-2))$

$= h(2) = 8$ $= h(6) = 20$.

Ex: $f(x) = \frac{1}{x-4}$ and $g(x) = \frac{2}{x} + 4$

Find $f \circ g$ and $g \circ f$.

Sep 20-12:22 PM

Decompose a composite function.

Eg let $h(x) = (x-5)^2$.

Find functions f and g such that

$f(g(x)) = h(x)$

Sol: * $f(x) = x^2$; $g(x) = x - 5$

* $f(x) = x$; $g(x) = (x-5)^2$.

Sep 20-12:30 PM

$h(x) = 4 + \sqrt[3]{x}$

Find 2 functions f and g such that

$f(g(x)) = h(x)$.

* $f(x) = x$; $g(x) = 4 + \sqrt[3]{x}$.
* $f(x) = 4 + x$; $g(x) = \sqrt[3]{x}$

$f(g(x)) = f(\sqrt[3]{x}) = 4 + \sqrt[3]{x}$

Domain of Composite functions.

Sep 20-12:34 PM

Strategy for finding the domain of $f(g(x))$.

Step 1: Find the domain of g.

Step 2: Find the domain of f

Step 3: Exclude the inputs x from the domain of g for which $g(x)$ is NOT in the domain of f. The result is the domain of $f \circ g$.

E.g. $f(x) = \frac{5}{x-1}$; $g(x) = \frac{4}{3x-2}$

Find the domain of $f \circ g$. Domain of g: all real #'s except for $\frac{2}{3}$

Sep 20-12:38 PM

Domain of f: all real #'s except for 1

$g(x) = 1$

$\frac{4}{3x-2} = 1$.

$4 = 3x - 2$

$6 = 3x$

$x = 2$

Exclude: $\frac{2}{3}, 2$ from the domain of $f \circ g$.

Domain of $f \circ g$ in interval notation: $(-\infty, \frac{2}{3}) \cup (\frac{2}{3}, 2) \cup (2, \infty)$.

Sep 20-12:43 PM