Zero-Product Property

If $A \cdot B = \boxed{0}$; then either $A = 0$ or $B = 0$

E.g 1: Factor and Solve the quadratic equation

$$x^2 - 5x - 6 = 0$$

$1 \cdot (-6) = -6$
$= 2 \cdot 3 = (-2) \cdot 3$
$= \boxed{(-6) \cdot 1}$

$$\underbrace{(x-6)}_{A}\underbrace{(x+1)}_{B} = 0$$

Either $x - 6 = 0$ OR $x + 1 = 0$

$\boxed{x = 6}$; $\boxed{x = -1}$

E.g. 2. Factor and solve the equation:

$$\boxed{12}x^2 + 11x + 2 = 0$$

$24 = \boxed{8 \cdot 3}$

$$\underbrace{12x^2 + 8x} + \underbrace{3x + 2} = 0$$

$4x\boxed{(3x+2)} + \boxed{(3x+2)} = 0$

$(3x+2)(4x+1) = 0$

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Either $4x + 1 = 0$ or $3x + 2 = 0$

$\boxed{x = -\frac{1}{4}}$; $\boxed{x = -\frac{2}{3}}$

E.g.3 $x^2 - 25 = 0$

$(x+5)(x-5) = 0$

$x + 5 = 0$ or $x - 5 = 0$

$\boxed{x = -5}$; $\boxed{x = 5}$.

Using the Square root Property

The square root property: $\boxed{\text{If } A^2 = B; \text{ then } A = \pm\sqrt{B}}$

$x^2 = 4$; $x \stackrel{?}{=}$

E.g. $4x^2 + 1 = 6$; $4x^2 = 5$; $x^2 = \frac{5}{4}$

By the Square Root Property: $x = \pm\sqrt{\frac{5}{4}} = \pm\frac{\sqrt{5}}{\sqrt{4}} = \boxed{\pm\frac{\sqrt{5}}{2}}$

E.g. $3(x-4)^2 = 15$; $(x-4)^2 = 5$; $x - 4 = \pm\sqrt{5}$

$\boxed{x = 4 \pm \sqrt{5}}$

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Completing the Square

$\boxed{x^2 + 4x + 1 = 0}$

$1 = 1 \cdot 1$
$= (-1) \cdot (-1)$

(1) $\boxed{x^2 + 4x = -1}$;

(2) Multiply the b term by $\frac{1}{2}$ and square it.

$\frac{1}{2} \cdot 4 = 2.$ $2^2 = 4$

(3) Add the number you get from Step (2) to both sides of equation 1

$$\underbrace{x^2 + 4x + 4} = -1 + 4$$

$(x+2)^2 = 3$; $x + 2 = \pm\sqrt{3}$

$\boxed{x = -2 \pm \sqrt{3}}$.

E.g. Solve the equation $x^2 - 3x - 5 = 0$

(1) $x^2 - 3x = 5$

(2) $-3 \cdot \frac{1}{2} = -\frac{3}{2}$ $\left(-\frac{3}{2}\right)^2 = \frac{9}{4}$.

(3) $\underbrace{x^2 - 3x + \frac{9}{4}} = 5 + \frac{9}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$

$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \frac{\pm\sqrt{29}}{2}$

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$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2} = \frac{3 \pm \sqrt{29}}{2}$.

$x^2 - 6x - 13 = 0.$

(1) $x^2 - 6x = 13$

(2) $(-6) \cdot \frac{1}{2} = -3$; $(-3)^2 = 9$

(3) $\underbrace{x^2 - 6x + 9} = 13 + 9$

$(x-3)^2 = 22$

$x - 3 = \pm\sqrt{22}$

$\boxed{x = 3 \pm \sqrt{22}}$.

Quadratic Formula:

The solutions to the quadratic equation $ax^2 + bx + c = 0$ are given by: $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$; $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

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E.g. Solve the equation

$$9x^2 + 3x - 2 = 0$$

Using the quadratic formula.

$a = 9$; $b = 3$; $c = -2$.

$$x_1 = \frac{-3 + \sqrt{(3)^2 - 4 \cdot 9 \cdot (-2)}}{2 \cdot 9}$$

$x_1 = \frac{-3 + \sqrt{81}}{18}$; $x_2 = \frac{-3 - \sqrt{81}}{18}$

$x_1 = \frac{-3 + 9}{18}$; $x_2 = \frac{-3 - 9}{18}$

$x_1 = \frac{6}{18} = \boxed{\frac{1}{3}}$; $x_2 = \frac{-12}{18} = \boxed{-\frac{2}{3}}$

E.g. $x^2 + x + 1 = 0$; $a = 1$; $b = 1$; $c = 1$

$x_1 = \frac{-1 + \sqrt{(1)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 + \sqrt{-3}}{2} = \frac{-1 + i\sqrt{3}}{2}$

$x_2 = \frac{-1 - i\sqrt{3}}{2}$.

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Why the quadratic formula is true?

$$ax^2 + bx + c = 0$$

(1) $ax^2 + bx = -c$

Divide both sides by a

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

(2) $\frac{b}{a} \cdot \frac{1}{2} = \frac{b}{2a}$, $\left(\frac{b}{2a}\right)^2 = \boxed{\frac{b^2}{4a^2}}$

(3) $\underbrace{x^2 + \frac{b}{a}x + \boxed{\frac{b^2}{4a^2}}} = -\frac{c \cdot 4a}{a \cdot 4a} + \frac{b^2}{4a^2}$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$; $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

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$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\boxed{x=\frac{-b\pm\sqrt{\boxed{b^2-4ac}}}{2a}}$$

The quantity b^2-4ac is called the discriminant of the quadratic expression ax^2+bx+c.

Discriminant of a quadratic expression

$b^2-4ac>0$: $ax^2+bx+c=0$ has 2 real solutions

$b^2-4ac<0$: $ax^2+bx+c=0$ has 2 complex solutions (no real solutions)

$b^2-4ac=0$: $ax^2+bx+c=0$ has 1 solution. $x=\frac{-b}{2a}$.

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Eg. $3x^2-5x-2=0$

Use the discriminant to determine the # of solutions & types of solution of this equation.

Discriminant: $b^2-4ac=(-5)^2-4\cdot(3)\cdot(-2)$
$=25+24=49>0$

It has 2 real solutions.

$x^2+6x+9=0$

Discriminant $=6^2-4\cdot1\cdot9=36-36=0$

It has 1 solution.

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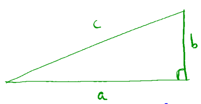
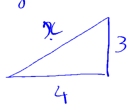
Pythagorean Theorem



$a^2+b^2=c^2$

E.g. If a right triangle has 2 legs leg a measured 4 units, leg b measured 3 units. Find the length of the hypotenuse.



$3^2+4^2=x^2$

$x^2=25$

$x=5$

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