

2.6 - Other types of equations

- Goals:
- ① Solve equations with rational exponents.
 - ② Solve polynomial equations by grouping and factoring.
 - ③ Solve radical equations
 - ④ Solve equations that involve absolute values.
 - ⑤ Solve equations in quadratic form using the substitution method.
 - ⑥ Some application problems.

Equations with Rational Exponents

Rational Exponent. $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4.$$

$$\frac{1}{\frac{1}{3}} \text{ recd: } \left(\sqrt[3]{8}\right)^2 = 2^2 = 4.$$

$$64^{\frac{1}{3}} = \frac{1}{64^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}.$$

General formula:

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

E.g. 1: Solve an equation that has a variable raised to a rational exponent.

$$x^{\frac{3}{2}} = 125$$

Reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$. Raise both sides to the exponent $\frac{2}{3}$.

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$$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(125\right)^{\frac{3}{2}}; \quad x^{\frac{2}{3} \cdot \frac{3}{2}} = \left(125\right)^{\frac{3}{2}}$$

$$x = \left(\sqrt[3]{125}\right)^2; \quad x = 5^2; \quad \boxed{x = 25}.$$

E.g. $(x+5)^{\frac{3}{2}} = 8$

$$\left((x+5)^{\frac{3}{2}}\right)^{\frac{2}{3}} = \left(8\right)^{\frac{2}{3}}$$

$$x+5 = 4; \quad \boxed{x = -1}$$

E.g. $3x^{\frac{3}{4}} = x^{\frac{1}{4}}; \quad 3x^{\frac{3}{4}} - x^{\frac{1}{4}} = 0$

$$x^{\frac{1}{4}}(3x^{\frac{3}{4}} - 1) = 0$$

Zero product property

Either $x^{\frac{1}{4}} = 0$ or $3x^{\frac{3}{4}} - 1 = 0$

$$x^{\frac{1}{4}} = 0 \Rightarrow x = 0$$

$$\left(x^{\frac{1}{4}}\right)^4 = 0^4 \Rightarrow \boxed{x = 0}$$

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$$3x^{\frac{1}{4}} - 1 = 0; \quad 3x^{\frac{1}{4}} = 1; \quad x^{\frac{1}{4}} = \frac{1}{3}$$

$$\left(x^{\frac{1}{4}}\right)^4 = \left(\frac{1}{3}\right)^4; \quad x = \left(\frac{1}{3}\right)^4; \quad \boxed{x = \frac{1}{81}}$$

Solve Polynomial equations by factoring

E.g. $12x^4 = 3x^2; \quad 12x^4 - 3x^2 = 0$

$$3x^2(4x^2 - 1) = 0; \quad \text{Either } 3x^2 = 0 \text{ or } 4x^2 - 1 = 0$$

$$3x^2 = 0; \quad x^2 = 0; \quad \boxed{x = 0}$$

$$4x^2 - 1 = 0; \quad 4x^2 = 1; \quad x^2 = \frac{1}{4}$$

$$\boxed{x = \pm \frac{1}{2}}$$

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Solve polynomial equation by grouping & factoring.

E.g. $(x^3 + 2x^2 - x - 2) = 0$

$$x^2(x+2) - (x+2) = 0$$

$$(x+2)(x^2 - 1) = 0$$

Either $x+2=0$ or $x^2 - 1 = 0$

$$\boxed{x = -2} \quad x^2 = 1; \quad \boxed{x = \pm 1}$$

Ex: $5x^3 + 45x = 2x^2 + 18$

$$5x^3 + 45x - 2x^2 - 18 = 0$$

$$5x(x^2 + 9) - 2(x^2 + 9) = 0$$

$$(5x - 2)(x^2 + 9) = 0$$

$$5x - 2 = 0 \Rightarrow \boxed{x = \frac{2}{5}}$$

$$(x^2 + 9)(5x - 2) = 0$$

Either $x^2 + 9 = 0$ or $5x - 2 = 0$

$$x^2 + 9 = 0 \Rightarrow x^2 = -9 \Rightarrow x = \pm \sqrt{-9} \Rightarrow x = \pm 3i$$

(No real solutions)

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Radical Equation.

E.g. $\sqrt{x+3} = 3x-1$

Square both sides:

$$\left(\sqrt{x+3}\right)^2 = (3x-1)^2$$

$$x+3 = 9x^2 - 6x + 1$$

$$0 = 9x^2 - 7x - 2$$

$$9x^2 - 7x - 2 = 0 \quad -18 = (-9) \cdot 2 \quad \text{check own answer.}$$

$$9x(x-1) + 2(x-1) = 0$$

$$(x-1)(9x+2) = 0 \quad \text{Either } x-1=0 \text{ or } 9x+2=0$$

$$\boxed{x = 1}; \quad \boxed{x = -\frac{2}{9}}$$

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Check $x=1$ ✓: $\sqrt{x+3} = 3x-1$
 $\sqrt{1+3} = 3 \cdot 1 - 1$
 $\sqrt{4} = 2$ ✓

Check $x = -\frac{2}{5}$ ✗
 $\sqrt{x+3} = 3x-1$
 $\sqrt{-\frac{2}{5}+3} = 3 \cdot (-\frac{2}{5}) - 1$
 $\sqrt{\frac{13}{5}} = -\frac{6}{5} - 1 = -\frac{11}{5}$
 $\frac{\sqrt{13}}{\sqrt{5}} \neq -\frac{11}{5}$
 extraneous solution.

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2. $\sqrt{3x+7} + \sqrt{x+2} = 1$
 $\sqrt{3x+7} = 1 - \sqrt{x+2}$
 Square both sides
 $(\sqrt{3x+7})^2 = (1 - \sqrt{x+2})^2$
 $3x+7 = (1 - \sqrt{x+2})(1 + \sqrt{x+2})$
 $3x+7 = 1 - x - 2\sqrt{x+2}$
 $2x+4 = -2\sqrt{x+2}$
 $(2x+4)^2 = (-2\sqrt{x+2})^2$
 $4x^2 + 16x + 16 = 4(x+2)$
 $x^2 + 4x + 4 = x+2$
 $x^2 + 3x + 2 = 0$
 $(x+2)(x+1) = 0$
 $x = -2, x = -1$

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Original equation:
 $\sqrt{3x+7} + \sqrt{x+2} = 1$
 Check: $x = -2$ ✓
 $\sqrt{3(-2)+7} + \sqrt{-2+2} = 1$
 $\sqrt{1} + \sqrt{0} = 1$
 $1 + 0 = 1$ ✓
 Check: $x = -1$ ✗
 $\sqrt{3(-1)+7} + \sqrt{-1+2} = 1$
 $\sqrt{4} + \sqrt{1} = 1$
 $2 + 1 = 1$ ✗

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Solve absolute value equations
 E.g. $|x| = 7$; $x = \pm 7$
 $|x| = -7$; No solution.
 $|x| = 0$; $x = 0$
 E.g. $|1-4x| + 8 = 13$
 $|1-4x| = 5$ Either $1-4x = 5$; $x = -1$
 or $1-4x = -5$; $x = \frac{3}{2}$
 $|ax+b| = c$. If $c > 0$; then either $ax+b = c$
 or $ax+b = -c$
 If $c < 0$, no solution. If $c = 0$; $ax+b = 0$

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Equations quadratic in form (substitution method)
 E.g. $x^4 - 8x^2 - 9 = 0$
 let $u = x^2$
 $u^2 - 8u - 9 = 0$
 $(u-9)(u+1) = 0$
 $u = 9$ or $u = -1$
 If $u = 9$, then $x^2 = 9$
 $x = \pm 3$ — real solutions
 If $u = -1$, then $x^2 = -1$
 $x = \pm i$
 E.g. $(x^2-1)^2 + (x^2-1) - 12 = 0$
 let $u = x^2-1$
 $u^2 + u - 12 = 0$
 $(u+4)(u-3) = 0$
 $u = -4$ or $u = 3$
 If $u = -4$, then $x^2-1 = -4$
 $x^2 = -3$
 $x = \pm i\sqrt{3}$
 If $u = 3$, then $x^2-1 = 3$
 $x^2 = 4$
 $x = \pm 2$

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