3.1 - Functions and Function Notation.

Goals
1. Determine whether a relation represents a function.
2. Evaluate a function at a given input.
3. Apply vertical line test to determine whether a graph represents a function.
4. Apply horizontal line test to determine whether a graph represents a one-to-one function.

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Relations and Functions

A relation is a set of ordered pairs

Ordered pair: $(1,2) \neq (2,1)$: $(m,n) \neq (n,m)$

(Plain Donut, \$1.49) $\neq$ (\$1.49, Plain Donut)

Set: collection of objects

E.g. of a relation: $\{(1,2);(2,4);(3,6);(4,8);(5,10)\}$

$\{(a,b);(c,d);(e,f);(g,h)\}$

What is a function?

$\{(1,2);(2,4);(3,6);(4,8);(5,10)\}$

Domain of a relation: set of first components

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Domain = $\{1,2,3,4,5\}$ elements of domain are called input

Range of a relation: set of second components.

Range = $\{2,4,6,8,10\}$ elements of range are called outputs

E.g. Relation $\{(odd,1);(even,2);(odd,3);(even,4)\}$

Domain = $\{odd, even\}$

Range = $\{1,2,3,4\}$

Definition of a function: A function is a relation that assigns a single element in the range to each element in the domain.

E.g. $\{(1,2);(2,4);(3,6);(4,8);(5,10)\}$

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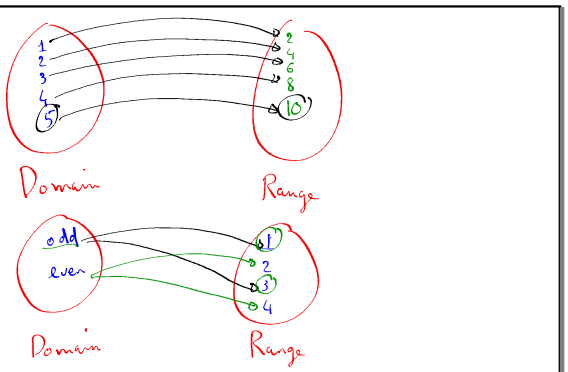

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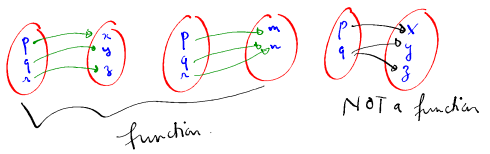
E.g. $\{(p,x);(q,y);(r,z)\}$ → a function

$\{(p,m);(q,m);(r,n)\}$ → a function

$\{(p,x);(q,y);(q,z)\}$ → NOT a function



Function:

A function is a relation in which each input value leads to exactly one output value. We say that the output is a function of the input.

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The input values make up the domain, the output values make up the range.

E.g.

Menu	
Item	Price
Plain Donut	1.49
Jelly Donut	1.99
Chocolate Donut	1.99

(1) Is Price a function of the item? Yes

(2) Is the item a function of the price? No.

Function Notation

E.g. Function whose input is the name of a month and output is the # of days in that month.

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{(January, 31), (February, 28), (March, 31) ....}.

days = function of month

days = f(month)

f(January) = 31

f(December) = 31.

The notation $y = f(x)$ defines a function named f.

This is read as y is a function of x. (output, input)

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Represent a function using Tables

Months	January	February	March	April ....
days	31	28	31	30 ...

E.g. Table 1:

Input	output
2	1
5	3
8	6

Table 2

Input	output
-3	5
0	1
4	5

Table 3

Input	output
1	0
5	2
5	4

f(5) = 3 function g(-3) = 5 Not a function

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$f(x) = -x^2 + 5$ ← algebraic form of a function

$\{(x, -x^2+5)\}$

(1,4); (2,1); (0,5)

$f(x) = x^2 + 3x - 4$.

$f(0) = -4$

$f(a) = a^2 + 3a - 4$

$f(a+1) = (a+1)^2 + 3(a+1) - 4$

$= a^2 + 2a + 1 + 3a + 3 - 4$

$f(a+1) = a^2 + 5a$

$f(a+h) = (a+h)^2 + 3(a+h) - 4$

$= a^2 + 2ah + h^2 + 3a + 3h - 4$

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$$\frac{f(a+h) - f(a)}{h} = \frac{a^2+2ah+h^2+3a+3h-4-(a^2+3a-4)}{h}$$

$$= \frac{a^2+2ah+h^2+3a+3h-4-a^2-3a+4}{h}$$

$$= \frac{2ah+h^2+3h}{h} = \frac{h(2a+h+3)}{h}$$

$= 2a + h + 3.$

E.g: $g(m) = \sqrt{m-4}$. (a) Evaluate g(5)

(b) Solve g(m) = 2.

(a) $g(5) = \sqrt{5-4} = \sqrt{1} = 1$

(b) $\sqrt{m-4} = 2$; $m - 4 = 4$; $m = 8$

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E.g. Find an equation for the function.

$2n + 6m = 12$.

Write this relation as a function $m = f(n)$?

$2n + 6m = 12$

$6m = 12 - 2n$; $m = \frac{12-2n}{6}$; $m = 2 - \frac{n}{3}$; $f(n)$

E.g: $x - 8y^3 = 0$.

Express y as a function of x.

$-8y^3 = -x$; $y^3 = \frac{x}{8}$; $y = \sqrt[3]{\frac{x}{8}}$

$y = \frac{\sqrt[3]{x}}{\sqrt[3]{8}}$; $y = \frac{\sqrt[3]{x}}{2}$.

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Find Function Values from a graph ✓

Vertical Line Test

If we can draw a vertical line which intersects a graph more than once, then the graph does not represent a function.

One-to-one functions.

A one-to-one function is a function in which each output value corresponds to exactly one input value.

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Horizontal line test:

If a horizontal line intersects the graph more than once, then the graph does not represent a one-to-one function.

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