

3.2 - Domain and Range

- Goals:
- ① Find the domain of a function defined by a formula.
 - ② Find domain and the range of a function given its graph.
 - ③ Evaluate and sketch the graph of a piecewise function.

Function given by a set of ordered pairs.

$$\{(-5, 4); (0, 0); (5, -4); (10, -8); (15, -12)\}$$

$$\text{Domain} = \{-5, 0, 5, 10, 15\}$$

$$\text{Range} = \{4, 0, -4, -8, -12\}$$

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Domain of a function given by a formula

$$f(x) = \frac{1}{x}$$

$$(x, \frac{1}{x})$$

$$(\frac{1}{1}, 1); (2, \frac{1}{2}); (-5, -\frac{1}{5})$$

$$(0, \frac{1}{0})$$

Process for finding domain of a function given by a formula

① Identify the input values.

② Identify any restrictions on the input and exclude those values from the domain.

③ Write the domain in interval notation.

E.g. $f(x) = x^2 - 1$. Find the domain.

No restriction on the input.

Domain = set of real numbers.

$$(-\infty, \infty)$$

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E.g. $f(x) = 5 - x + x^3$. Find the domain.

Any restriction on the input?

Domain = set of real numbers.

$$\text{Domain} = (-\infty, \infty)$$

E.g. $f(x) = \frac{x+1}{2-x}$. Find the domain.

Restriction: $x \neq 2$.

$$\text{Domain} = (-\infty, 2) \cup (2, \infty)$$

E.g. $f(x) = \frac{1+4x}{2x-1}$. Find the domain.

Restriction on the input? $2x-1=0$
 $x = \frac{1}{2}$

Domain in interval notation: $= (-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

E.g. $f(x) = \frac{x^2+6}{x^2+5x+6}$. Find domain?

Restriction on input: $x^2+5x+6=0$

$$(x+3)(x+2)=0$$

$$\text{Domain} = (-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$$

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$$f(x) = \frac{3x^2 + x + 12}{(x-1)(x-2)(x+3)(x^2+x-12)(x^2+1)}$$

Find the domain.

Any restriction? $(x-1)(x-2)(x+3)(x^2+x-12)(x^2+1) = 0$

$x-1=0; \boxed{x=1}$
 $x-2=0; \boxed{x=2}$
 $x+3=0; \boxed{x=-3}$

$x^2+x-12=0$
 $(x+4)(x-3)=0$
 $\boxed{x=-4}; \boxed{x=3}$

$x^2+1=0$
 $x^2 = -1$
 $x = \pm\sqrt{-1} = \pm i$

Zero product property

No real solutions

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$$\text{Domain} = (-\infty, -4) \cup (-4, -3) \cup (-3, 1) \cup (1, 2) \cup (2, 3) \cup (3, \infty)$$

Find the domain of a function with an even root

E.g. $f(x) = \sqrt{7-x}$. Find the domain.

Any restriction? $7-x \geq 0$

$x \leq 7$

Domain: $(-\infty, 7]$

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Find the domain of $f(x) = \sqrt{5+2x}$

Restriction? $5+2x \geq 0$

$2x \geq -5$
 $x \geq -\frac{5}{2}$

Domain: $[-\frac{5}{2}, \infty)$

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Find the domain and range from graphs.

Toolkit functions:

- Constant function: $f(x) = c$
c: constant.
- $f(x) = x$
- $f(x) = |x|$
- $f(x) = x^2$
- $f(x) = x^3$
- $f(x) = \frac{1}{x}$
- $f(x) = \frac{1}{x^2}$

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⑧ $f(x) = \sqrt{x}$ ⑨ $f(x) = \sqrt[3]{x}$

Piecewise Functions

A piecewise function is a function in which more than one formula is used to define the output. Each formula has its own domain. The domain of the function is the union of all the smaller domains.

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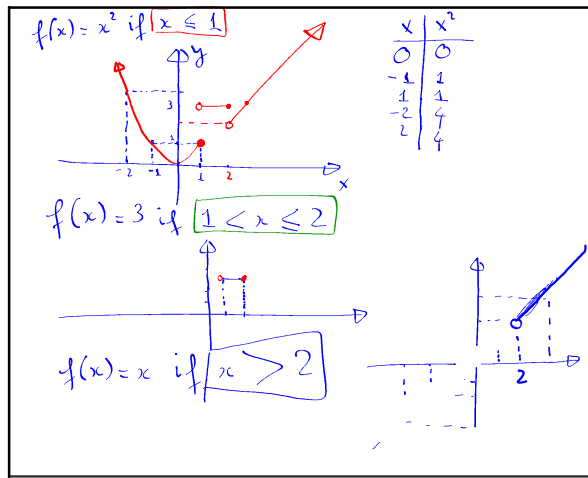
$$f(x) = \begin{cases} \text{formula 1} & \text{if } x \text{ is in domain 1} \\ \text{formula 2} & \text{if } x \text{ is in domain 2} \\ \text{formula 3} & \text{if } x \text{ is in domain 3} \end{cases}$$

E.g.

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 3 & \text{if } 1 < x \leq 2 \\ x & \text{if } x > 2 \end{cases}$$

$f(-1) = (-1)^2 = 1$
 $f(0) = 0^2 = 0$
 $f(2016) = 2016$
 $f(1.999) = 3$
 $f(2) = 3$

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