3.3 - Rates of Change and Behavior of graphs of functions

Goals: (1) Find the average rate of change of a given function and solve rated problems.

(2) Use the graph of a function to determine the intervals of increasing / decreasing, local max / min, absolute max / min.

Average Rate of change.

Gasoline cost.

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Average Rate of Change = $\dfrac{\text{Change in output}}{\text{Change in input}}$

$$= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

If the function is f, then : $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

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Average rate of change between 2 input values x_1 and x_2 $= \dfrac{\Delta y}{\Delta x} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

* Calculate the average rate of change of a function given by a table of values
* Calculate the average rate of change of a function given by a graph.
* Calculate the average rate of change of a function given by a formula.

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E.g. $\boxed{f(x) = x^2 - \frac{1}{x}}$.

Calculate the average rate of change of f on $[2,4]$.

Average rate of change $= \dfrac{f(4) - f(2)}{4 - 2}$

$$= \frac{\left(16 - \frac{1}{4}\right) - \left(4 - \frac{1}{2}\right)}{2} = \frac{\frac{63}{4} - \frac{7}{2}}{2} = \frac{\frac{49}{4}}{2} = \boxed{\frac{49}{8}}.$$

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Eg $f(x) = x - 2\sqrt{x}$.

Find the average rate of change of f on $\boxed{[1, 9]}$

Average rate of change $= \dfrac{f(9) - f(1)}{9 - 1}$

$$= \frac{(9 - 2\sqrt{9}) - (1 - 2\sqrt{1})}{8} = \frac{3 - (-1)}{8} = \frac{4}{8} = \boxed{\frac{1}{2}}.$$

E.g. $f(x) = x^2 + 2x - 8$.

Average rate of change of f on $\boxed{[5, a]}$

Average rate of change $= \dfrac{f(a) - f(5)}{a - 5}$

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$$\frac{f(a) - f(5)}{a - 5} = \frac{(a^2 + 2a - 8) - (5^2 + 2\cdot 5 - 8)}{a - 5}$$

$$= \frac{a^2 + 2a - 8 - (27)}{a - 5}$$

$$= \frac{a^2 + 2a - 35}{a - 5} = \frac{(a+7)\cancel{(a-5)}}{\cancel{a-5}}$$

Average rate of change of f on $[5, a]$ $= \boxed{a + 7}$

E.g. $f(x) = \frac{1}{x}$. Find the average rate of change of f on $[x, x+h]$.

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Average rate of change on $[x, x+h]$ $= \dfrac{f(x+h) - f(x)}{(x+h) - x}$.

$$= \frac{f(x+h) - f(x)}{h}$$

$$\boxed{\frac{-1}{x(x+h)}}$$

$$= \frac{\dfrac{1 \cdot x}{(x+h) \cdot x} - \dfrac{1 \cdot (x+h)}{x \cdot (x+h)}}{h}$$

$$= \frac{\dfrac{x}{x(x+h)} - \dfrac{x+h}{x(x+h)}}{h}$$

$$= \frac{\dfrac{x-(x+h)}{x(x+h)}}{h} = \frac{\dfrac{-h}{x(x+h)}}{h} = \frac{-h}{x(x+h)} \cdot \frac{1}{h}$$

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Behavior of the graph of a function.

* Increasing / Decreasing

f is decreasing on an interval if for any a, b in that interval: $a < b$ implies $f(a) > f(b)$

f is increasing on an interval if for any a, b in that interval: $a < b$ implies $f(a) < f(b)$

f has a local minimum at $x = a$ if f changes from being decreasing to being increasing after x passes a.

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f has a local maximum at $x = a$ if f changes from being increasing to being decreasing after x passes a.

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