6.3. Logarithmic Functions

E.g. $f(x) = 2^x$; $y = 2^x$

$2^{\boxed{3}} = \boxed{8}$; $\log_2 8 = 3$

$2^{-1} = \frac{1}{2}$; $\log_2 \frac{1}{2} = -1$

$2^5 = 32$; $\log_2 32 = 5$

Definition of a logarithmic function:

$b > 0$; $b \neq 1$; $x > 0$.

$\log_b x$ gives us the exponent y such that $b^y = x$

In other words, $\underbrace{\log_b x}_{\text{this is read as log base b of } x} = y$ is equivalent to $b^y = x$

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E.g. $\log_3(27) = 3$ (b/c $3^3 = 27$)

$\log_3\left(\frac{1}{9}\right) = -2$ (b/c $3^{-2} = \frac{1}{9}$)

Ex. Evaluate the following:

(1) $\log_{10}(100) = 2$ (2) $\log_{10}(1{,}000{,}000) = 6$

(3) $\log_{10}(0.1) = -1$ (4) $\log_{10}(0.0001) = -4$

(5) $\log_5(25) = 2$ (6) $\log_5\left(\frac{1}{5}\right) = -1$

(7) $\log_5(\sqrt{5}) = \frac{1}{2}$ (8) $\log_5(\sqrt[3]{5}) = \frac{1}{3}$

(9) $\log_5(\sqrt[4]{5^7}) = \frac{7}{4}$.

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Common logarithms

$\log_{10}(x)$ is denoted $\log(x)$ (read as log of x or common log of x)

E.g. $\log(100) = 2$; $\log(1000) = 3$; $\log(0.01) = -2$

Natural logarithms

$\log_e(x)$ is denoted by $\ln(x)$ (read as ln of x or natural log of x)

$e \approx 2.7182...$

E.g. $\ln(e) = 1$; $\ln(e^2) = 2$; $\ln(e^3) = 3$

$\ln(1) = 0$ (In general, $\log_b(1) = 0$ for any base b).

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Convert equations from logarithmic form to exponential form and vice versa.

E.g. Convert the given equation to exponential form.

$\log_{\boxed{a}}(b) = c$ ← logarithmic form

$a^c = b$ ← exponential form.

E.g. Convert the given equation to logarithmic form.

$10^a = b$ ← exponential form

$\log b = a$.

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Ex: (1) Convert to ~~logarithmic~~ exp. form. (b) $\ln(w) = n$.

(a) $\log_y(x) = -11$

(2) Convert to ~~exponential~~ log. form. (b) $n^4 = 103$

(a) $e^k = h$

(1)(a) $y^{-11} = x$ (b) $e^n = w$

(2)(a) $\ln(h) = k$ (b) $\log_n(103) = 4$.

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