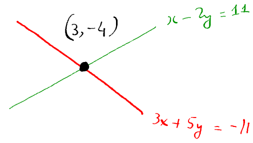
7.1. System of linear equations: in 2 variables

E.g.

$$\begin{cases} 3x + 5y = -11 \\ \boxed{x - 2y = 11} \end{cases}$$



An example of a system of 2 linear equations in 2 variables in x and y.

Substitution method: Choose an equation and solve for one variable in terms of the other.

$x - 2y = 11$; $\boxed{x = 11 + 2y}$

* Substitute x in terms of y into the other equation.

$3(11 + 2y) + 5y = -11$.

Dec 1-10:00 AM

$33 + 6y + 5y = -11$

$11y = -44$; $\boxed{y = -4}$

Since $x = 11 + 2y$; $x = 11 + 2\cdot(-4) = 11 - 8 = 3$

Solution to the system: $(3, -4)$
$\uparrow x \quad \uparrow y$

This is an example of a system with a unique solution. (When a system does have a solution, it is called a consistent system)

Example of a consistent system with infinitely many solutions

$$-3\begin{cases} (y - 2x) = 5\cdot(-3) \\ -3y + 6x = -15 \end{cases} \qquad \begin{cases} \boxed{-3y + 6x = -15} \\ \boxed{-3y + 6x = -15} \end{cases}$$

$\boxed{0 = 0}$

Elimination method.

Dec 1-10:08 AM

$-3y + 6x = 15$

$y = -5 + 2x$

Solution: x could be any # and $y = -5 + 2x$

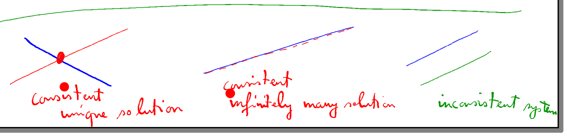
An example of an inconsistent system.

$$2\begin{cases} (y - x) = 1\cdot 2 \\ 2y - 2x = 6 \end{cases} \longrightarrow \begin{matrix} \cancel{2y} - \cancel{2x} = 2 \\ \cancel{2y} - \cancel{2x} = 6 \end{matrix}$$

Elimination

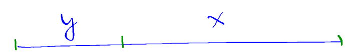
$0 = -4$ ← contradiction

No solution.



Dec 1-10:18 AM

#5: let x and y represent the lengths of the 2 pieces



$$\begin{cases} x + y = 10 \\ x - y = 8 \end{cases} \qquad 2y = 2 \qquad \boxed{y = 1} \qquad \boxed{x = 9}$$

#17. Cost of the turkey sandwich: x

salad: y

$$\begin{cases} 4x + 2y = 25 \\ 4(x - y) = 1\cdot 4 \end{cases} \longrightarrow \begin{matrix} \cancel{4x} + 2y = 25 \\ \cancel{4x} - 4y = 4 \end{matrix}$$

$6y = 21$

$y = \frac{21}{6}$ $\boxed{= 3.5}$

$\boxed{x = 4.5}$

Dec 1-10:26 AM

#20: Speed of boat in calm water: x

Speed of current: y

$x + y = \frac{20}{2} = 10$

$x - y = \frac{20}{5} = 4$

Dec 1-10:44 AM