

1.3 Rates of Change and Behavior of graphs

Goals:

- Find the average rate of change of a function.
- Determine the intervals of increasing/decreasing. Determine local/absolute minima/maxima of a function given its graph.

Average Rate of change = $\frac{\text{change in output}}{\text{change in the input}}$

Function: f

(x_1, y_1)
 (x_2, y_2)
 $y_1 = f(x_1)$
 $y_2 = f(x_2)$

$$\Delta y = y_2 - y_1 = f(x_2) - f(x_1)$$

$$\Delta x = x_2 - x_1$$

$$\text{Average Rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

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Average rate of change of f on $[x_1, x_2]$ = $\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

E.g. $f(x) = 4x^2 - 7$

Find the average rate of change of f on $[1, b]$

$$\frac{f(b) - f(1)}{b - 1} = \frac{4b^2 - 7 - (-3)}{b - 1}$$

$$= \frac{4b^2 - 4}{b - 1} = \frac{4(b^2 - 1)}{b - 1} = \frac{4(b-1)(b+1)}{b-1} = 4(b+1)$$

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$a(t) = \frac{1}{t+4}$

Find the average rate of change of a on $[9, 9+h]$

$$\frac{a(9+h) - a(9)}{(9+h) - 9} = \frac{\frac{1}{13+h} - \frac{1}{13}}{13+h - 13}$$

$$= \frac{\frac{13 - (13+h)}{(13+h) \cdot 13}}{h} = \frac{\frac{-h}{13(13+h)}}{h} = \frac{-1}{13(13+h)}$$

② Intervals of Increasing/Decreasing
 local minima / maxima.

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$f(x)$ is increasing if $b < a$ implies that $f(b) < f(a)$

$f(x)$ is decreasing if $b < a$ implies that $f(b) > f(a)$

E.g. $f(x) = \frac{1}{x}$ Find the number b such that the average rate of change of f on the interval $[2, b]$ is $-\frac{1}{10}$.

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Average rate of change of f on $[2, b]$

$$= \frac{f(b) - f(2)}{b - 2} = \frac{\frac{1}{b} - \frac{1}{2}}{b - 2} = -\frac{1}{10}$$

$$\frac{\frac{1}{b} - \frac{1}{2}}{b - 2} = -\frac{1}{10}$$

$$\frac{2-b}{2b} = -\frac{1}{10}(b-2)$$

$$\frac{2-b}{2b} = -\frac{1}{10}(b-2) = -\frac{1}{10}b + \frac{1}{5}$$

$$2-b = -\frac{1}{5}b^2 + \frac{2b}{5}$$

$$-\frac{1}{5}b^2 + \frac{2b}{5} + b - 2 = 0$$

$$-\frac{1}{5}b^2 + \frac{7b}{5} - 2 = 0$$

$$-b^2 + 7b - 10 = 0$$

$$b^2 - 7b + 10 = 0$$

$$(b-2)(b-5) = 0$$

$$b = 2, b = 5$$

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$$\frac{\frac{2-b}{2b}}{b-2} = -\frac{1}{10}$$

$$\frac{-(b-2)}{2b} = -\frac{1}{10}$$

$$\frac{1}{2b} = \frac{1}{10}$$

$$2b = 10$$

$$b = 5$$

Composition of functions $f(x) = 2x+1$; $g(x) = 3-x^2$

$$f \circ g(x) = f(g(x)) = 2 \cdot (3-x^2) + 1 = 6 - 2x^2 + 1 = -2x^2 + 7$$

$$g \circ f(x) = g(f(x)) = 3 - (2x+1)^2 = 3 - (2x+1)^2$$

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