

3.2 - Quadratic Functions (Mostly a Review)

- ① Use the standard form of a quadratic function to find the vertex, the max/min value, the axis of symmetry, the domain and range of the function.
- ② Sketch the graph of a quadratic function. Find the general form of the equation of a quadratic function given its graph.
- ③ Solve application problems that involve quadratic functions.

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① The Standard form of a quadratic function.

$$f(x) = ax^2 + bx + c \quad (\text{general form})$$

E.g. $f(x) = 3x^2 - 10x + 1$; $f(x) = -7x^2 - 6x + 11$

Standard form: $f(x) = a(x-h)^2 + k$

E.g. Given $f(x) = x^2 + 4x + 3$.

Find the standard form of f ?

Completing the square?

Sum of a square formula.
Difference

$$(b+a)^2 = x^2 + 2 \cdot a \cdot x + a^2$$

$$(x+a)^2 = x^2 + 2ax + a^2$$

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$$x^2 + 4x + 3 = x^2 + 2 \cdot 2x + 4 - 4 + 3$$

$$= (x+2)^2 - 1$$


$$f(x) = x^2 + 4x + 3$$

$$\rightarrow f(x) = (x+2)^2 - 1 = (x - (-2))^2 - 1$$

In the standard form $a(x-h)^2 + k$ for f .
What is h ; what is k ? $h = -2$; $k = -1$

Vertex of the parabola: $(-2, -1)$

Reason why $(-2, -1)$ is the vertex: $(x+2)^2 \geq 0$; $(x+2)^2 - 1 \geq -1$

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$$f(x) = 3x^2 - 6x - 9 \quad (\text{in general form})$$

Use the method of completing the square to turn f into the standard form $f(x) = a(x-h)^2 + k$.

$$f(x) = 3(x^2 - 2x) - 9$$

$$= 3(x^2 - 2x + 1 - 1) - 9$$

$$= 3((x-1)^2 - 1) - 9$$

$$= 3(x-1)^2 - 3 - 9 = 3(x-1)^2 - 12$$

$$f(x) = 3(x-1)^2 - 12$$

$a = 3$; $h = 1$; $k = -12$ Vertex: $(1, -12)$.

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In general, if $f(x) = ax^2 + bx + c$
then in the standard form $f(x) = a(x-h)^2 + k$

$$h = -\frac{b}{2a}; k = c - \frac{b^2}{4a}$$

$$f(x) = a(x-h)^2 + k$$

$$x = h \quad f(h) = a(h-h)^2 + k = k$$

$$h = f(h)$$

$$f(x) = 3x^2 - 6x - 9; h = \frac{6}{6} = 1; k = f(1) = -12$$

Standard form: $f(x) = 3(x-1)^2 + (-12)$
 $= 3(x-1)^2 - 12$

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Let $f(x) = x^2 + 5x - 2$

Find the standard form of f ?

$$f(x) = a(x-h)^2 + k$$

$$a = 1; h = -\frac{5}{2}; k = f\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^2 + 5 \cdot \left(-\frac{5}{2}\right) - 2$$

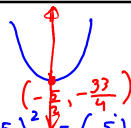
$$f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{33}{4}$$

Vertex: $\left(-\frac{5}{2}, -\frac{33}{4}\right)$

Minimum of f : $-\frac{33}{4}$

Axis of symmetry: $x = -\frac{5}{2}$

Domain: $(-\infty, \infty)$
Range: $\left[-\frac{33}{4}, \infty\right)$



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$h(t) = -4t^2 + 6t - 1$
 ① Find the standard form $h(t) = a(x-h)^2 + k$ of h .
 $a = -4$; $h = \frac{-b}{2a} = \frac{-6}{2(-4)} = \frac{3}{4}$;
 $k = f\left(\frac{3}{4}\right) = -4\left(\frac{3}{4}\right)^2 + 6\left(\frac{3}{4}\right) - 1$
 $= -4 \cdot \frac{9}{16} + \frac{18}{4} - 1$
 $= -\frac{9}{4} + \frac{9}{2} - 1 =$
 $= -\frac{9}{4} + \frac{18}{4} - 1 = \boxed{\frac{5}{4}}$
 $f(x) = -4\left(x - \frac{3}{4}\right)^2 + \frac{5}{4}$
 Vertex: $\left(\frac{3}{4}, \frac{5}{4}\right)$ Domain: $(-\infty, \infty)$
 Maximum: $\frac{5}{4}$ Range: $(-\infty, \frac{5}{4}]$
 Axis of symmetry: $x = \frac{3}{4}$

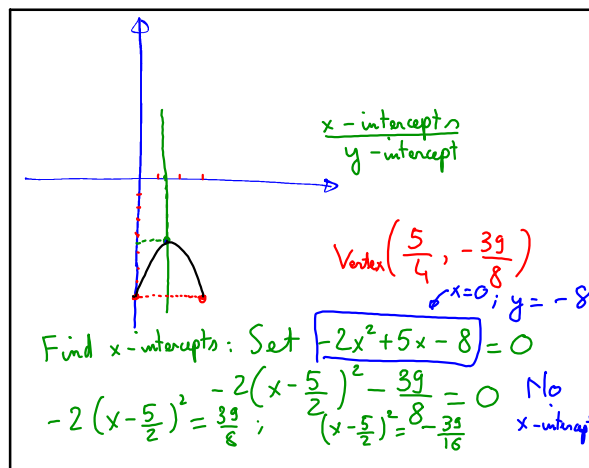
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$f(x) = ax^2 + bx + c$
 Standard form: $f(x) = a(x-h)^2 + k$
 where $h = -\frac{b}{2a}$; $k = f\left(-\frac{b}{2a}\right)$
 Vertex: (h, k)
 If $a > 0$, f has a minimum. $\text{Min } f = k$
 If $a < 0$, f has a maximum. $\text{Max } f = k$
 Domain: $(-\infty, \infty)$
 Range: $\begin{cases} a > 0: [k, \infty) \\ a < 0: (-\infty, k] \end{cases}$
 Axis of symmetry: $x = h$

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Ex 57/3.2
 $f(x) = -2x^2 + 5x - 8$
 Sketch a graph of f by using the standard form to find the vertex, axis of symmetry & maximum.
 Find x-intercepts, y-intercept, and extra points on the graph if necessary.
 $f(x) = a(x-h)^2 + k$
 $h = \frac{-b}{2a} = \frac{5}{4}$; $k = f\left(\frac{5}{4}\right)$
 $k = -2\left(\frac{5}{4}\right)^2 + 5 \cdot \frac{5}{4} - 8$
 $= -2 \cdot \frac{25}{16} + \frac{25}{4} - 8$
 $= -\frac{25}{8} + \frac{25}{4} - 8 = \frac{-25 + 50 - 64}{8}$
 $= \frac{-39}{8}$

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To find the x -intercept of the graph of $f(x)$.
 Set $f(x) = 0$
 To find the y -intercept of $f(x)$ we set $x=0$ in the formula for f
 Find the equation (in general form) of a quadratic function using its graph.

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