4.5. Logarithmic Properties

Goals: (1) Apply the logarithmic properties to condense or expand expressions.

(2) Apply the change of base formula to solve problems

$\log_2(2^3) = 3$

* Inverse Properties.

$\log_b(b^x) = x$

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$b^{\log_b x} = x$ $\quad 2^{\log_2 4} = 4$

$\boxed{\log_b(b^x) = x \\ b^{\log_b(x)} = x}$

Product Rule for Logarithms

$\boxed{\log_b(M \cdot N) = \log_b(M) + \log_b(N)}$

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$\log_2(4 \cdot x) = \log_2(4) + \log_2(x)$

$\log_2(4x) = 2 + \log_2(x)$

E.g. Use the product rule to expand the expression $\boxed{\log_3(30x(x+4))}$

$\log_3(\underbrace{\boxed{30x}}_{M}\underbrace{\boxed{(x+4)}}_{N}) = \boxed{\log_3(30x)} + \log_3(x+4)$

$= \log_3(30) + \log_3(x) + \log_3(x+4)$

$= \log_3(3 \cdot 10) + \log_3(x) + \log_3(x+4)$

$= \log_3(3) + \log_3(10) + \log_3(x) + \log_3(x+4)$

$= 1 + \log_3(10) + \log_3(x) + \log_3(x+4)$

Why is the product rule true?

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$\log_b(MN) = \underbrace{\log_b(M)}_{x} + \underbrace{\log_b(N)}_{y}$

RHS: let $\boxed{x = \log_b(M)}$; $\boxed{b^x = M}$

$\boxed{y = \log_b(N)}$; $\boxed{b^y = N}$

RHS $\log_b(\boxed{b^x \cdot b^y}) = \log_b(b^{x+y}) = x + y.$

RHS $= x + y$

RHS $= x + y$

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Quotient Rule:

$\boxed{\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)}$

$\log\left(\frac{100}{y}\right) = \log(100) - \log(y)$

$= 2 - \log(y)$

$\neq \boxed{\log(100 - y)}$

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E.g. $\log_3\left(\frac{7x^2 + 21x}{7x(x-1)(x-2)}\right)$

Expand this expression.

$= \log_3\left(\frac{\cancel{7x}(x+3)}{\cancel{7x}(x-1)(x-2)}\right)$

$= \log_3\left(\frac{x+3}{(x-1)(x-2)}\right) = +\log_3(x+3) - \log_3(x-1) - \log_3(x-2)$

$= \log_3(x+3) - \boxed{\log_3((x-1)(x-2))}$

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$= \log_3(x+3) - \left[\log_3(x-1) + \log_3(x-2)\right]$

$= \boxed{\log_3(x+3) - \log_3(x-1) - \log_3(x-2)}$

$\log_3\left(\frac{A \cdot B \cdot C}{D \cdot E \cdot F}\right) = \log_3(A) + \log_3(B) + \log_3(C) - \log_3(D) - \log_3(E) - \log_3(F)$

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Ex: $\log_2\left(\frac{\boxed{15 \cdot x}(x-1)}{(3x+4)(2-x)}\right)$

Expand this expression in one stroke.

$\log_2(15) + \log_2(x) + \log_2(x-1)$

$\log_2(3(x+4)) - \log_2((3x)+(4)) - \log_2(2-x)$

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Power Rule for logarithms:

$\log_b(M^2) = \log_b(M \cdot M)$

$= \log_b(M) + \log_b(M)$

$\boxed{\log_b(M^2) = 2\log_b(M)}$

$\log_b(M^3) = \log_b(M^2 \cdot M)$

$= \log_b(M^2) + \log_b(M)$

$= 2\log_b(M) + \log_b(M)$

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$\log_b(M^3) = 3\log_b(M)$

$\log_b(M^{2016}) = 2016 \cdot \log_b(M)$

Power Rule for Log

$\boxed{\log_b(M^p) = p \cdot \log_b(M)}$

$\log_2(x^5) = 5\log_2(x)$

$\ln(e^7) = 7\ln(e) = 7$

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Expand: $\ln\left(\frac{1}{x^2}\right) = \ln(x^{-2})$

$= -2\ln(x)$.

$\log(\sqrt{x}) = \log(x^{\frac{1}{2}}) = \frac{1}{2}\log(x)$

$\ln(\sqrt[3]{x^2}) = \ln(x^{\frac{2}{3}}) = \frac{2}{3}\ln(x)$.

Ex: Expand $\ln\left(\frac{\sqrt{(x-1)(2x+1)^3}}{x^2-9}\right)$

$\frac{1}{2}\ln(x-1) + \frac{3}{2}\ln(2x+1) - \ln(x+3) - \ln(x-3)$

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$\ln\left(\frac{((x-1)(2x+1)^3)^{1/2}}{(x+3)(x-3)}\right)$

$\ln\left((x-1)(2x+1)^3\right)^{1/2} - \ln(x+3) - \ln(x-3)$

$\frac{1}{2}\ln\left((x-1)(2x+1)^3\right) - \ln(x+3) - \ln(x-3)$

$\frac{1}{2}\ln(x-1) + \frac{3}{2}\ln(2x+1) - \ln(x+3) - \ln(x-3)$

$\log_b(MN) = \log_b(M) + \log_b(N)$

$\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

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$\log_b(M) + \log_b(N) = \log_b(MN)$

$\log_b(M) - \log_b(N) = \log_b\left(\frac{M}{N}\right)$

E.g. Write $\log_3(5) + \log_3(8) - \log_3(2)$ as a single logarithm.

$(\log_3(5) + \log_3(8)) - \log_3(2)$

$\log_3(40) - \log_3(2) = \log_3(20)$

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$\log_3(\boxed{5}) + \log_3(\boxed{8}) - \log_3(\boxed{2})$

$= \log_3\left(\frac{5 \cdot 8}{2}\right) = \log_3(20)$

Condense:

$1 \cdot \log(3) - \log(4) + \log(5) - \log(6) + \log(7) - \log(8)$

$= \log\left(\frac{\cancel{3} \cdot 5 \cdot 7}{4 \cdot \cancel{6} \cdot 8}\right) = \log\left(\frac{35}{64}\right)$

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E.g. Condense

$\log_2(x^2) + \boxed{\frac{1}{2}}\log_2(x-1) - \boxed{3}\log_2(x+3)^2$

$\log_2(x^2) + \log_2(x-1)^{\frac{1}{2}} - \log_2(x+3)^6$

$$\boxed{\log_2\left(\frac{x^2 \cdot \sqrt{x-1}}{(x+3)^6}\right)}$$

Change of base formula

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$\boxed{\log_2 3} = \frac{\ln 3}{\ln 2}$

$$\boxed{\log_{\boxed{a}} \boxed{b} = \frac{\log_c b}{\log_c a}}$$

Change of base formula.

$\log_{\boxed{2}} 3 = \frac{\log_e 3}{\log_e 2} = \frac{\ln 3}{\ln 2}$

$\parallel$

$\frac{\log 3}{\log 2}$

$\boxed{\text{base } e}$

base 10

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