

4.6 Exponential and Logarithmic Equations

Goals: ① Solve Exp. & Log Equations
② Solve Application Problems.

① Using like bases to solve Exp. equations

E.g. $5^{2x} = 5^{3x+2}$
 $2x = 3x + 2$; $x = -2$

Sep 14-7:26 PM

Rewrite Equations so that all powers have the same base.

E.g. $5^{2x} = (25)^{3x+2}$
 $5^{2x} = (5^2)^{3x+2}$
 $5^{2x} = 5^{2(3x+2)}$
 $2x = 2(3x+2)$
 $x = 3x + 2$; $x = -1$

Sep 14-7:29 PM

E.g.

$$5^{-3x+5} = \sqrt{5}$$

$$5^{-3x+5} = 5^{\frac{1}{2}}$$

$$-3x+5 = \frac{1}{2} ; -3x = -\frac{9}{2}$$

$$-3x = \frac{1}{2} - 5 ; x = \frac{3}{2}$$

E.g. $5^{x+2} = 4^x$

Sep 14-7:31 PM

Solve exponential equations using logarithms.

E.g. $5^{x+2} = 4^x$

Take the ln of both sides.

$$\ln(5^{x+2}) = \ln(4^x)$$

Power Rule for Log

$$(x+2) \ln(5) = x \ln(4)$$

$$x \cdot \ln(5) + 2 \cdot \ln(5) = x \cdot \ln(4)$$

$$\begin{aligned} x \cdot \ln(5) - x \cdot \ln(4) &= -2 \ln(5) \end{aligned}$$

Sep 14-7:33 PM

$$x \cdot \ln(5) - x \cdot \ln(4) = -2 \ln(5)$$

$$x (\ln(5) - \ln(4)) = -2 \ln(5)$$

$$x \cdot \ln\left(\frac{5}{4}\right) = -2 \ln(5)$$

$$x = \frac{-2 \ln(5)}{\ln\left(\frac{5}{4}\right)}$$

Sep 14-7:41 PM

Some equations containing e

E.g. Solve $3e^{0.5x} = 11$

$$e^{0.5x} = \frac{11}{3}$$

$$\ln(e^{0.5x}) = \ln\left(\frac{11}{3}\right)$$

$$0.5x = \ln\left(\frac{11}{3}\right)$$

$$x = \frac{\ln\left(\frac{11}{3}\right)}{0.5}$$

Sep 14-7:44 PM

$$3 + e^{2t} = 7e^{2t} \quad \text{Solve for } t.$$

$$3 = 7e^{2t} - e^{2t}$$

$$3 = 6e^{2t}$$

$$\frac{1}{2} = e^{2t}$$

$$e^{2t} = \frac{1}{2}$$

$$\ln(e^{2t}) = \ln\left(\frac{1}{2}\right)$$

$$2t = \ln\left(\frac{1}{2}\right)$$

$$t = \frac{\ln\left(\frac{1}{2}\right)}{2}$$

Sep 14-7:48 PM

$$e^{2x} - e^x - 56 = 0$$

$$(e^x)^2 - e^x - 56 = 0$$

$$\text{let } e^x = A$$

$$A^2 - A - 56 = 0$$

$$(A-8)(A+7) = 0$$

$$A = 8 \text{ or } A = -7$$

$$e^x = 8$$

$$x = \ln 8$$

$$\text{or}$$

$$e^x = -7$$

$$x = \ln(-7)$$

Sep 14-7:53 PM

Logarithmic Equations

Solve by using the definition of logarithm.

E.g. $6 + \ln(x) = 10$

$$\ln(x) = 4$$

$$x = e^4$$

$$2\ln(x+1) = 10$$

$$\ln(x+1) = 5$$

$$x+1 = e^5$$

$$x = e^5 - 1$$

Sep 14-8:00 PM

Using Properties of log to solve the equation.

E.g. $\ln(x^2) = \ln(2x+3)$

$$x^2 = 2x+3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3; x = -1$$

$$\ln(9) = \ln(9)$$

$$\ln(1) = \ln(1)$$

Sep 14-8:03 PM

$$\ln(-3x) = \ln(x^2 - 6x)$$

$$-3x = x^2 - 6x$$

$$0 = x^2 - 3x$$

$$x(x-3) = 0$$

$$x = 0$$

$$x = 3$$

Sep 14-8:07 PM

E.g. $\ln(x) + \ln(x-3) = \ln(7x)$

$$\ln(x(x-3)) = \ln(7x)$$

$$x(x-3) = 7x$$

$$x^2 - 3x = 7x$$

$$x^2 - 10x = 0$$

$$x(x-10) = 0$$

$$x = 0$$

$$\text{or } x = 10$$

Sep 14-8:09 PM