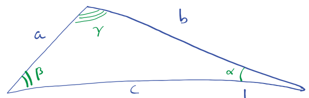
8.2. Law of Cosines (Cont.)



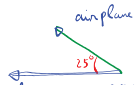
$a^2 = b^2 + c^2 - 2bc\cos\alpha$ | $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$b^2 = a^2 + c^2 - 2ac\cos\beta$ | $\cos\beta = \dfrac{a^2+c^2-b^2}{2ac}$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$ | $\cos\gamma = \dfrac{a^2+b^2-c^2}{2ab}$

#73.

airplane #2 : 420 mph



airplane #1 speed, 300 mph

After 90 minutes, how far apart are they?

1.5

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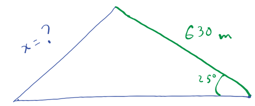
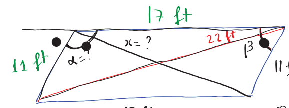


$x^2 = 450^2 + 630^2 - 2\cdot 450\cdot 630\cdot\cos(25°)$

$x = \sqrt{450^2 + 630^2 - 2\cdot 450\cdot 630\cdot\cos(25°)} = 292.44$

E.g.



$\cos\alpha = \dfrac{11^2 + 17^2 - 22^2}{2\cdot 11\cdot 17}$

$\alpha \simeq 101.4°$

$\beta = 180° - 101.4°$

$\simeq 78.6°$

$x^2 = 17^2 + 11^2 - 2\cdot 17\cdot 11\cdot\cos(78.6°)$

$x = \sqrt{}$

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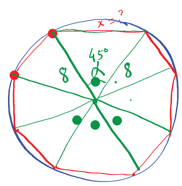
E.g.



Regular Octagon inscribed in a circle of radius 8 inches.

Find the perimeter of the octagon?

$8\alpha = 360°$

$\alpha = \dfrac{360°}{8} = 45°$

$x^2 = 8^2 + 8^2 - 2\cdot 8\cdot 8\cdot\cos(45°)$

$x^2 = 128 - 128\cdot\frac{\sqrt{2}}{2} = 128\left(1 - \frac{\sqrt{2}}{2}\right)$; $x = \sqrt{128\cdot\left(1-\frac{\sqrt{2}}{2}\right)}$

Perimeter $= 8x = 8\sqrt{128\cdot\left(1-\frac{\sqrt{2}}{2}\right)}$.

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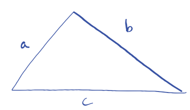
Heron's Formula

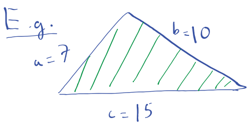


Area $= \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a+b+c}{2}$ (half perimeter)

Area = ?

E.g.



$s = \dfrac{7+10+15}{2} = 16$

Area $= \sqrt{16\cdot(16-7)(16-10)\cdot(16-15)}$

$= \sqrt{16\cdot 9\cdot 6\cdot 1} = 12\sqrt{6}$.

Why Heron's formula is true?

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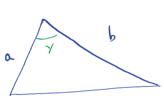

$\boxed{\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}}$

$\cos\gamma = \boxed{\dfrac{a^2+b^2-c^2}{2ab}}$

Area $= \frac{1}{2}ab\boxed{\sin\gamma}$

$\sin^2\gamma + \cos^2\gamma = 1$

$\sin^2\gamma = 1 - \cos^2\gamma$

$\sin\gamma = \sqrt{1-\cos^2\gamma}$

$\sin\gamma = \sqrt{1 - \left(\dfrac{a^2+b^2-c^2}{2ab}\right)^2}$

$\sin\gamma = \sqrt{1 - \dfrac{(a^2+b^2-c^2)^2}{4a^2b^2}} = \sqrt{\dfrac{4a^2b^2 - (a^2+b^2-c^2)^2}{4a^2b^2}}$

$\sin\gamma = \boxed{\dfrac{\sqrt{4a^2b^2 - (a^2+b^2-c^2)^2}}{2ab}}$

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Area $= \frac{1}{2}\cancel{ab}\cdot\dfrac{\sqrt{4a^2b^2 - (a^2+b^2-c^2)^2}}{2\cancel{ab}}$

$\boxed{A^2 - B^2 = (A-B)\cdot(A+B)}$

Area $= \frac{1}{4}\cdot\sqrt{(4a^2b^2) - (a^2+b^2-c^2)^2}$

$= \frac{1}{4}\cdot\sqrt{\underbrace{(2ab)}_{A}{}^2 - \underbrace{\boxed{a^2+b^2-c^2}}_{B}{}^2}$

$a^2+b^2+2ab = (a+b)^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$= \frac{1}{4}\sqrt{\left(2ab - (a^2+b^2-c^2)\right)\cdot\left(2ab + a^2+b^2-c^2\right)}$

$= \frac{1}{4}\sqrt{(-a^2-b^2+2ab+c^2)\cdot\left((a+b)^2 - c^2\right)}$

$= \frac{1}{4}\cdot\sqrt{\left(c^2-(a-b)^2\right)\cdot\left((a+b)^2 - c^2\right)}$

$= \frac{1}{4}\cdot\sqrt{(c-a+b)(c+a-b)(a+b-c)(a+b+c)}$

$\sqrt{2\cdot(s-a)\cdot 2\cdot(s-b)\cdot 2(s-c)\cdot 2s}$

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