8.7 Parametric Equations (Cont.)

Graphs of Parametric Equations: $0 \le x \le \sqrt{3}$

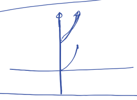
$x = f(t)$; $y = g(t)$

E.g. $\boxed{x = \sqrt{t}\ ;\ y = 2t + 3}$; $0 \le t \le 3$

t	x	y
0	0	3
$\frac{1}{2}$	$\sqrt{\frac{1}{2}}$	4

$x = \sqrt{t}$; $\boxed{t = x^2}$

$\boxed{y = 2x^2 + 3}$



E.g. $\boxed{\begin{array}{l} x = 2\cos t \\ y = 4\sin t \end{array}}$ $\quad \frac{x}{2} = \cos t$; $\frac{y}{4} = \sin t$; $\left.\begin{array}{l} \frac{x^2}{4} = \cos^2 t \\ \frac{y^2}{16} = \sin^2 t \end{array}\right\} +$

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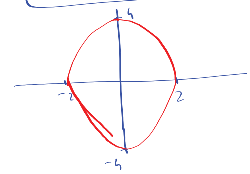
$\frac{x^2}{4} + \frac{y^2}{16} = \cos^2 t + \sin^2 t = 1$

$\boxed{\frac{x^2}{4} + \frac{y^2}{16} = 1}$ ellipse



Projectile Motion.

An object is thrown upward with an initial velocity v_0,
Angle between initial velocity and horizontal axis is θ

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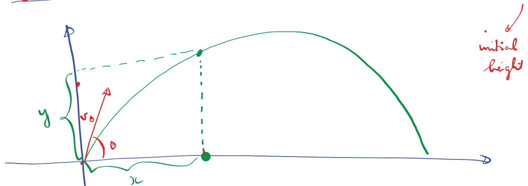
$g = 9.8\ m/s^2$ or $32\ ft/s^2$

$x = (v_0 \cos\theta)t$

$y = -\frac{1}{2}\boxed{g}t^2 + \boxed{(v_0 \sin\theta)}t + \boxed{h}$

initial height



$$\boxed{\begin{array}{l} x = (v_0 \cos\theta)t \\ y = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t + h \end{array}}$$

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E.g. A dart is thrown upward with an initial velocity of 140 ft/s at an angle 45° to the horizontal and at an initial height of 3 ft above the ground.

(1) Find the parametric equations to model the path of the dart

(2) Where is the dart after 2 seconds

(3) How long is the dart in the air

(4) At what time will the dart reach maximum height and find the maximum height.

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$v_0 = 140\ ft/s.$

$\theta = 45°$

$h = 3\ ft$

$x = (v_0 \cos\theta)t = (140\cos(45°))t = (70\sqrt{2})t$

$\boxed{x = 70\sqrt{2}\,t}$

$y = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t + h$

$y = -\frac{1}{2}\cdot 32 \cdot t^2 + (140\sin(45°))t + 3$

$\boxed{y = -16t^2 + 70\sqrt{2}\,t + 3}$

(2) When $\boxed{t = 2}$; $x = (70\sqrt{2})\cdot 2 = 140\sqrt{2} \approx$

$y = -64 + 140\sqrt{2} + 3 \approx$

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(3) Time in the air.

Solve $y = 0$

$-16t^2 + 70\sqrt{2}\,t + 3 = 0$

$t = \dfrac{-70\sqrt{2} \pm \sqrt{(70\sqrt{2})^2 - 4(-16)\cdot 3}}{2\cdot(-16)}$

$t = -0.03$

$\boxed{t = 6.22}$

$ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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y is maximum when $t = \frac{-b}{2a} = \frac{-70\sqrt{2}}{2\cdot(-16)} = \boxed{\frac{70\sqrt{2}}{32}} \approx$

function for y.

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