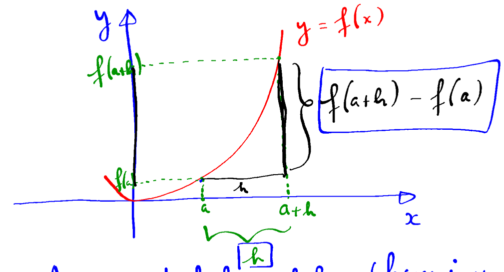
3.4 - Derivatives as Rates of Change.

Goals: Interpret the derivative as a rate of change and solve applications

1. Motion along a line.
2. Population Change
3. Cost and Revenue.

Sep 19-4:42 PM

Derivative as a rate of change.



$\boxed{h}$

Average rate of change of f on $[a, a+h]$ $= \dfrac{\text{change in } y}{\text{change in } x}$

Sep 19-4:45 PM

Average Rate of change of f on $[a, a+h]$ $= \dfrac{f(a+h) - f(a)}{h}$

Instantaneous rate of change of f at the point $x = a$ $= \boxed{\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}}$

$\boxed{\text{Inst. R.O.C. of } f \text{ at } x = a \;=\; f'(a)}$

Sep 19-4:48 PM

Motion along a line

$s(t)$: position of the object at time t.

$s(1) = 2$ at $t = 1$, the object is 2 units from the origin.

$v(t)$: velocity of the object at time t.

$\boxed{v(t) = s'(t)}$

$\boxed{v(t) > 0}$ means object is moving from $\boxed{\text{left to right.}}$

$\boxed{v(t) < 0}$ ——————— from right to left

$|v(t)|$: speed of the object at time t.

Sep 19-4:52 PM

$a(t)$: acceleration of the object at time t.

$\boxed{a(t) = v'(t) = s''(t)}$

$a(t) > 0$: object is speeding up

$a(t) < 0$: object is slowing down.

E.g. A particle moves along a line in the positive direction. $\boxed{3.35}$ Its position at time t is given by

$$s(t) = t^3 - 4t + 2.$$

(a) Find $v(1)$ and $a(1)$

(b) Is the particle moving from left to right or from right to left?

(c) Is the particle speeding up or slowing down?

Sep 19-4:56 PM

$v(t) = s'(t) = 3t^2 - 4$

$(s(t) = t^3 - 4t + 2)$

$a(t) = v'(t) = 6t$

$v(1) = -1$; $a(1) = 6$

$v(1) < 0$ (b) It moves from right to left.

$a(1) > 0$ (c) It is speeding up.

E.g. Position function: $s(t) = t^3 - 9t^2 + 24t + 4$; $t \geq 0$

(a) At what time(s) is the particle at rest?

(b) Find the intervals on which the particle is moving from left to right? right to left?

Sep 19-5:02 PM

(c) Find the intervals on which the particle is speeding up? slowing down?

$v(t) = \boxed{3t^2 - 18t + 24}$

Particle is at rest means $v(t) = 0$

$\boxed{3t^2 - 18t + 24} = 0$

$3(t-2)(t-4) = 0$

$\boxed{t=2}$; $\boxed{t=4}$

(b) From left to right: $v(t) > 0$; $\boxed{3t^2 - 18t + 24 > 0}$

From right to left: $v(t) < 0$; $\boxed{3t^2 - 18t + 24 < 0}$

Sep 19-5:05 PM

t	0	$t=1$	2	$t=3$	4	$t=5$
$v(t)$		$+$	0	$-$	0	$+$

$\boxed{3t^2 - 18t + 24} > 0$ at $t=1$

$\boxed{v(t) > 0}$ on $[0,2)$ and $(4,\infty)$

$\boxed{v(t) < 0}$ on $(2,4)$

it is moving from left to right on $[0,2)$ and $(4,\infty)$

it is moving from right to left on $(2,4)$.

(c) $a(t) = v'(t) = 6t - 18 = 0$; $t = 3$

t	0	$t=2$	3	$t=4$
$a(t)$		$-$	0	$+$

$a(t) > 0$ on $(3,\infty)$

$a(t) < 0$ on $(0,3)$

Sep 19-5:10 PM

Speeding up on: $(3,\infty)$

slowing down on: $[0,3)$.

Population change.

$P(t)$: # of entities in a population at time t.

$P'(t)$: growth rate of the population at t.

Ex 3.23: The current population of a mosquito colony is known to be 3000. $P(0) = 3000$

Sep 19-5:17 PM

If $P'(0) = 100$, estimate the size of the population in 3 days, t is measured in days. $P(3)$

$$\boxed{\frac{P(3) - P(0)}{3 - 0}} \approx P'(0)$$

$$\frac{P(3) - P(0)}{3} \approx P'(0)$$

$$P(3) - P(0) \approx 3P'(0)$$

$$P(3) \approx P(0) + 3\boxed{P'(0)} = \boxed{3300}$$

Sep 19-5:20 PM

$$\lim_{h \to 0} \boxed{\frac{f(a+h) - f(a)}{h}} = f'(a)$$

when h is small

$$\frac{f(a+h) - f(a)}{h} \approx f'(a)$$

$$\boxed{f(a+h)} \approx f(a) + h f'(a)$$

Sep 19-5:23 PM