3.6 - Part I

The Chain Rule.

$$\frac{d}{dx}\left(x^2\right) = 2x$$

$$\frac{d}{dx}\left(x^3\right) = 3x^2$$

$$\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x$$

$$\frac{d}{dx}\left(\left(\sin x + x + 5\right)^2\right) = ?$$

$$\frac{d}{dx}\left(\left(2x^2+x+7\right)^3\right) = ?$$

$$\frac{d}{dx}\left(\sqrt{\tan x}\right) = ?$$

$$\frac{d}{dx}\left(\frac{1}{x^3-x^2+\cos x}\right) = ?$$

$$\frac{d}{dx}\left(\sin(\csc x)\right) = ?$$

Sep 26-4:45 PM

Chain Rule for derivative.

If y is a function of u and u is a function of x, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

$$y \xrightarrow{\frac{dy}{du}} u \xrightarrow{\frac{du}{dx}} x$$

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$

Sep 26-4:52 PM

$$\frac{d}{dx}\left(\left(\underbrace{\sin x + x + 5}_{u}\right)^{\boxed{2}}\right)$$

$$y \xrightarrow{\frac{dy}{du}} u \xrightarrow{\frac{du}{dx}} x$$

Let $\boxed{u = \sin x + x + 5}$, $\frac{du}{dx} = \cos x + 1$

$\boxed{y = u^2}$ $\frac{dy}{du} = 2u$

$$\frac{dy}{dx} = \boxed{\frac{dy}{du}} \cdot \boxed{\frac{du}{dx}} = 2u \cdot (\cos x + 1)$$

$$= 2(\sin x + x + 5)\cdot(\cos x + 1)$$

Sep 26-4:56 PM

$$\frac{d}{dx}\left(\cos\left(\sin x\right)\right)$$

$$y \text{ — } u \text{ — } x$$

Let $u = \boxed{\sin x}$

$\boxed{y = \cos(u)}$

$$\frac{dy}{dx}$$

$$\frac{dy}{dx} = \boxed{\frac{dy}{du}} \cdot \frac{du}{dx} = -\sin(u)\cdot\cos x$$

$$= -\sin(\sin x)\cdot\cos x.$$

E.g. $\frac{d}{dx}\left(\sqrt{\boxed{\tan x}}\right)$ Let $\boxed{u = \tan x}$ $\frac{du}{dx} = \sec^2 x$

$\boxed{y = \sqrt{u}}$ $\frac{dy}{du} = \frac{1}{2\sqrt{u}}$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{2\sqrt{u}}\cdot\sec^2 x = \frac{1}{2\sqrt{\tan x}}\cdot\sec^2 x.$$

Sep 26-4:59 PM

Power Rule: $\frac{d}{dx}\left(\boxed{x}^n\right) = n x^{n-1}$

Trig Functions:

$$\frac{d}{dx}\left(\sin \boxed{x}\right) = \cos x$$

$$\frac{d}{dx}\left(\cos \boxed{x}\right) = -\sin x$$

$$\frac{d}{dx}\left(\tan \boxed{x}\right) = \sec^2 x$$

$$\frac{d}{dx}\left(\cot \boxed{x}\right) = -\csc^2 x$$

Let u be a function of x.

$$\frac{d}{dx}\left(\boxed{u}^n\right) = n u^{n-1}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(\sin(u)\right) = \cos(u)\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cos(u)\right) = -\sin(u)\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(\tan(u)\right) = \sec^2(u)\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cot(u)\right) = -\csc^2(u)\cdot\frac{du}{dx}$$

Sep 26-5:05 PM

$$\frac{d}{dx}\left(\boxed{\sec}\left(\boxed{4x^5+2x}\right)\right) = \sec(4x^5+2x)\cdot\tan(4x^5+2x)\cdot(20x^4+2)$$

$$\frac{d}{dx}\left(\cos\left(\boxed{5x^2}\right)\right) = -\sin(5x^2)\cdot(10x)$$

$$= -10x\sin(5x^2).$$

$$\frac{d}{dx}\left(\frac{x}{(2x+3)^3}\right) = \frac{d}{dx}\left(x\cdot(2x+3)^{-3}\right)$$

$$= 1\cdot(2x+3)^{-3} + x\cdot\boxed{\frac{d}{dx}\left(\left(\boxed{2x+3}\right)^{-3}\right)}$$

$$= (2x+3)^{-3} + x\cdot(-3)\cdot(2x+3)^{-4}\cdot 2$$

$$= (2x+3)^{-3} - 6x(2x+3)^{-4}.$$

Sep 26-5:09 PM

(2) In prime notation

$$\boxed{\left(f\left(\boxed{g}(x)\right)\right)' = f'(g(x)) \cdot g'(x)}.$$

Sep 26-5:13 PM