Note: f is a continuous function over a closed interval $[a,b]$

The absolute maximum/minimum of f can only occur at either the critical points of f in $[a,b]$ or at the endpoints a or b.

Method of finding abs. max/min of a continuous function over a closed interval $[a,b]$.

(1) Find all the critical #s (points) of f in $[a,b]$.

(2) Evaluate f at the points we found in (1)

(3) Evaluate f at the endpoints a and b

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(4) The largest value from (2) and (3) is the abs. max

the smallest value from (2) and (3) is the abs. min.

E.g. $f(x) = x^3 - 6x^2 + 9x + 1$ over $[0,5]$.

Find abs max/min of f over $[0,5]$

(1) Critical points: $f'(x) = 3x^2 - 12x + 9$

$f'(x) = 0$ when $3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$\boxed{x=1}$; $\boxed{x=3}$

(2) Evaluate f at critical points $f(1)$ and $f(3)$

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$f(1) = (1)^3 - 6(1)^2 + 9(1) + 1$

$\boxed{f(1) = 5}$

$f(3) = (3)^3 - 6(3)^2 + 9(3) + 1$

$= 27 - 54 + 27 + 1$

$\boxed{f(3) = 1}$

(3) Evaluate f at endpoints

$\boxed{f(0) = 1}$; $f(5) = (5)^3 - 6\cdot(5)^2 + 9\cdot 5 + 1$

$= 125 - 150 + 45 + 1$

$\boxed{f(5) = 21}$

(4) Abs. max = 21 when $x = 5$; Abs. min = 1; occurs when $x = 0, x = 3$

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E.g. $f(x) = x \cdot e^{-x^2/8}$; $[-1,4]$.

Find abs. max/min of f over $\boxed{[-1,4]}$

(1) Critical points.

$f'(x) = 1 \cdot e^{-x^2/8} + x \cdot e^{-x^2/8} \cdot \left(\frac{-2x}{8}\right)$

$f'(x) = \boxed{e^{-x^2/8}} - \frac{x^2}{4} \cdot \boxed{e^{-x^2/8}}$

$f'(x) = e^{-x^2/8}\left(1 - \frac{x^2}{4}\right)$

$\underbrace{\boxed{e^{-x^2/8}}}_{>0} \boxed{\left(1 - \frac{x^2}{4}\right)} = 0$ when $1 - \frac{x^2}{4} = 0$

$\frac{x^2}{4} = 1$; $x^2 = 4$; $\boxed{x = \pm 2}$

Only $\boxed{x=2}$ belongs to the interval $[-1,4]$

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(2) $f(2) = 2 \cdot e^{-4/8} = \boxed{2 \cdot e^{-1/2}} \approx \boxed{1.21}$

(3) $f(-1) = \boxed{-e^{-1/8}}$

$f(4) = 4 \cdot e^{-16/8} = \boxed{4 \cdot e^{-2}} \approx 0.54$

(4) Abs. min $= -e^{-1/8}$ it occurs when $\boxed{x = -1}$

Abs. max $= 2 \cdot e^{-1/2}$ it occurs when $\boxed{x = 2}$.

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