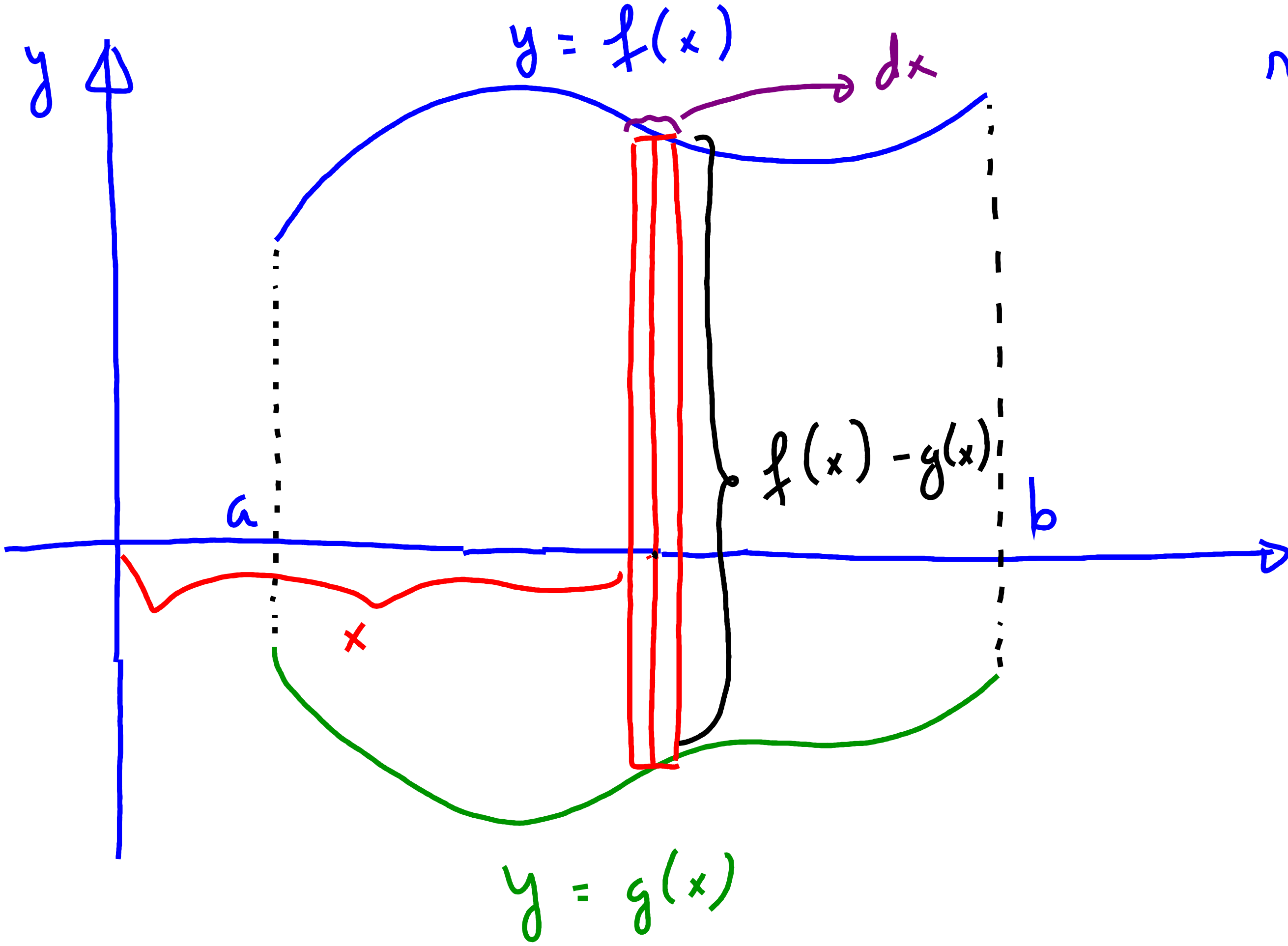
2.1. Areas between curves



Find the area of the region bounded by the curves $y = f(x)$; $y = g(x)$ between $a \leq x \leq b$

Area of a small rectangular strip is :

width $= dx$

Height $= f(x) - g(x)$

Area $= \left(f(x) - g(x)\right) \cdot dx$

Area we want = Infinite Sum of these little rectangular strips

$$= \boxed{\int_a^b} \left(f(x) - g(x) \right) dx$$

Sum

HW2.1 #1

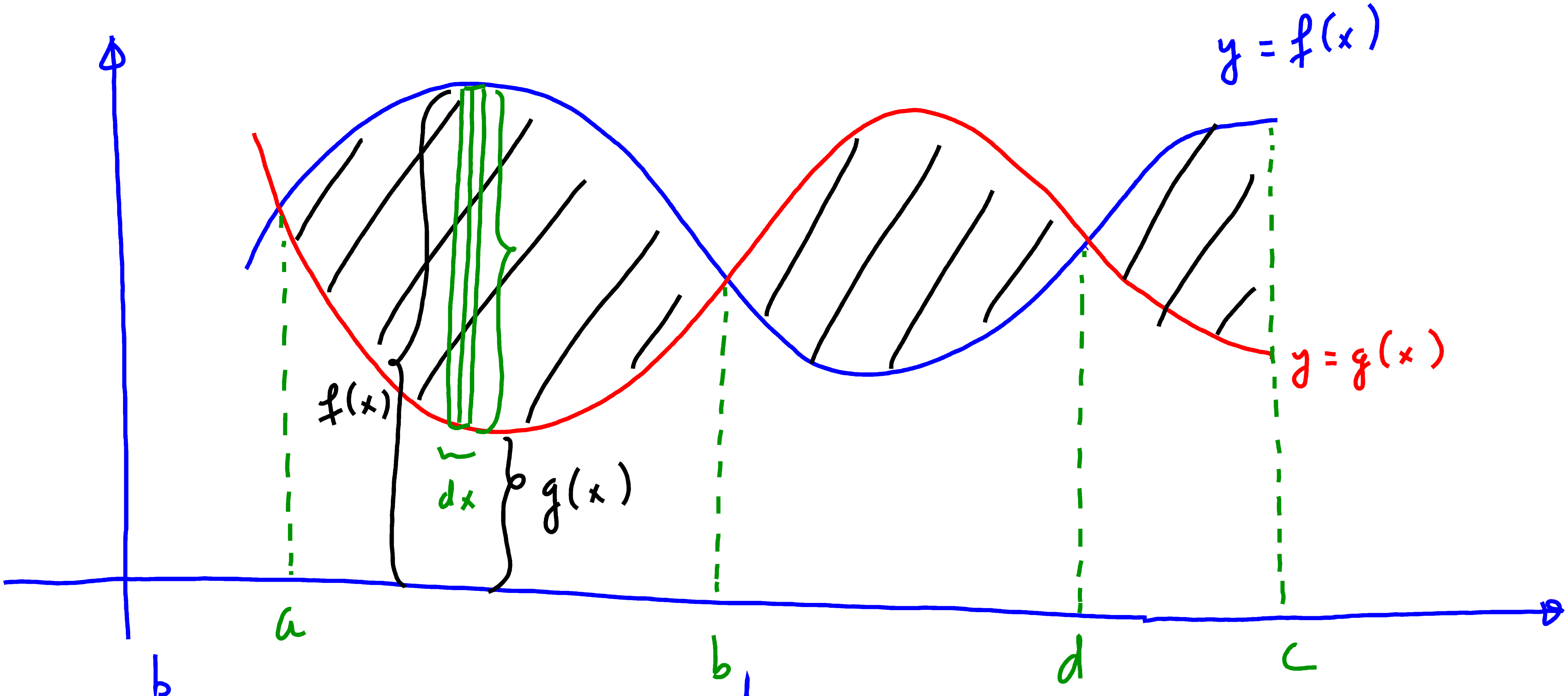
$$\int_{-\sqrt{3}}^{\sqrt{3}} \left(x^3 - 5x^2 - 3x + 15\right) dx$$

$$= \left(\frac{x^4}{4} - 5\cdot\frac{x^3}{3} - 3\cdot\frac{x^2}{2} + 15x\right)\Bigg|_{-\sqrt{3}}^{\sqrt{3}}$$

$$= \left(\cancel{\frac{(\sqrt{3})^4}{4}} \boxed{- 5\cdot\frac{(\sqrt{3})^3}{3}} \cancel{- 3\cdot\frac{(\sqrt{3})^2}{2}} + 15\cdot\sqrt{3}\right) - \left(\cancel{\frac{(-\sqrt{3})^4}{4}} - 5\cdot\frac{(-\sqrt{3})^3}{3} \cancel{- 3\,\frac{(-\sqrt{3})^2}{2}} - 15\cdot\sqrt{3}\right)$$

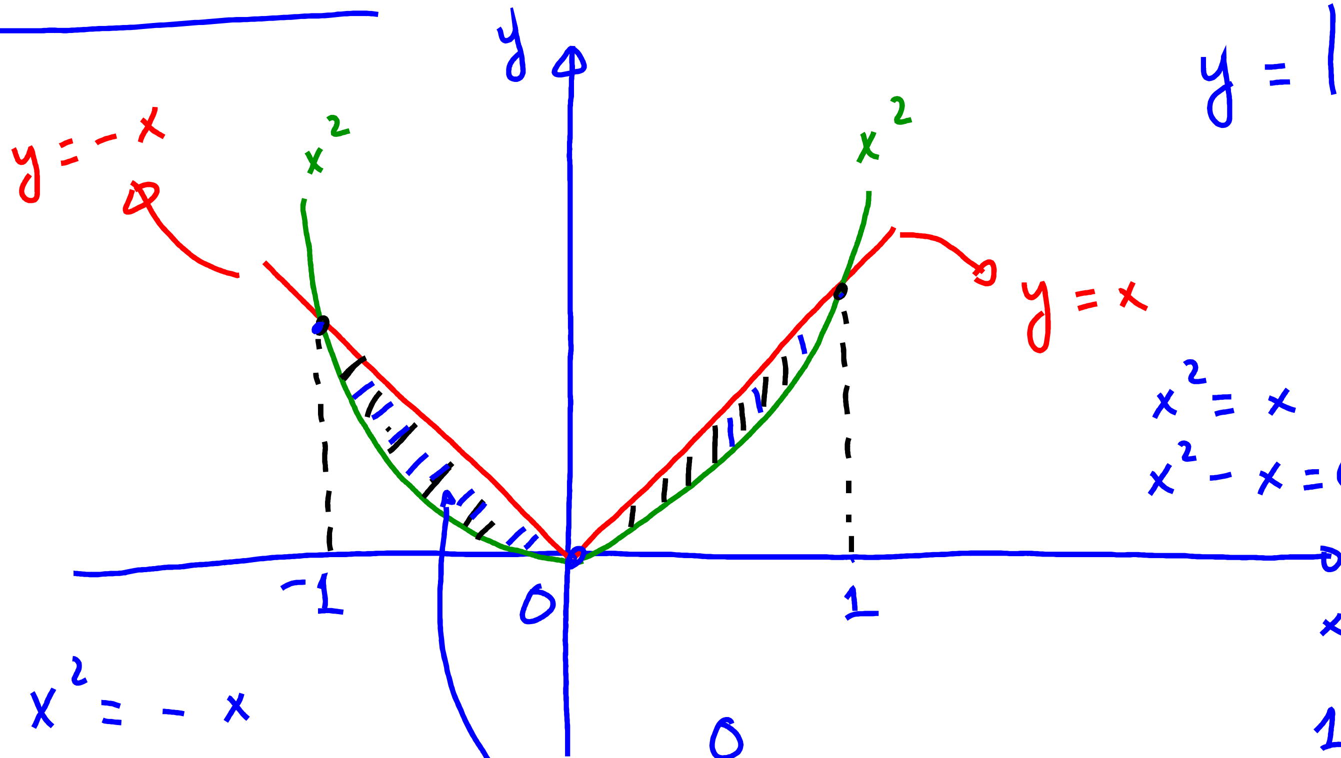
$$= -5\cdot\frac{3\sqrt{3}}{3} - \frac{5\cdot3\sqrt{3}}{3} + 15\sqrt{3} + 15\sqrt{3}$$

$$= -10\sqrt{3} + 30\sqrt{3} = \boxed{20\sqrt{3}}$$



$$\int_a^b \left(f(x) - g(x)\right) dx + \int_b^d \left(g(x) - f(x)\right) dx + \int_d^c \left(f(x) - g(x)\right) dx$$

Ex 8 (HW 1)



$$y = |x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

$x^2 = x$

$x^2 - x = 0$; $x(x-1)=0$

$x=0$; $x=1$

$x^2 = -x$

$x^2 + x = 0$

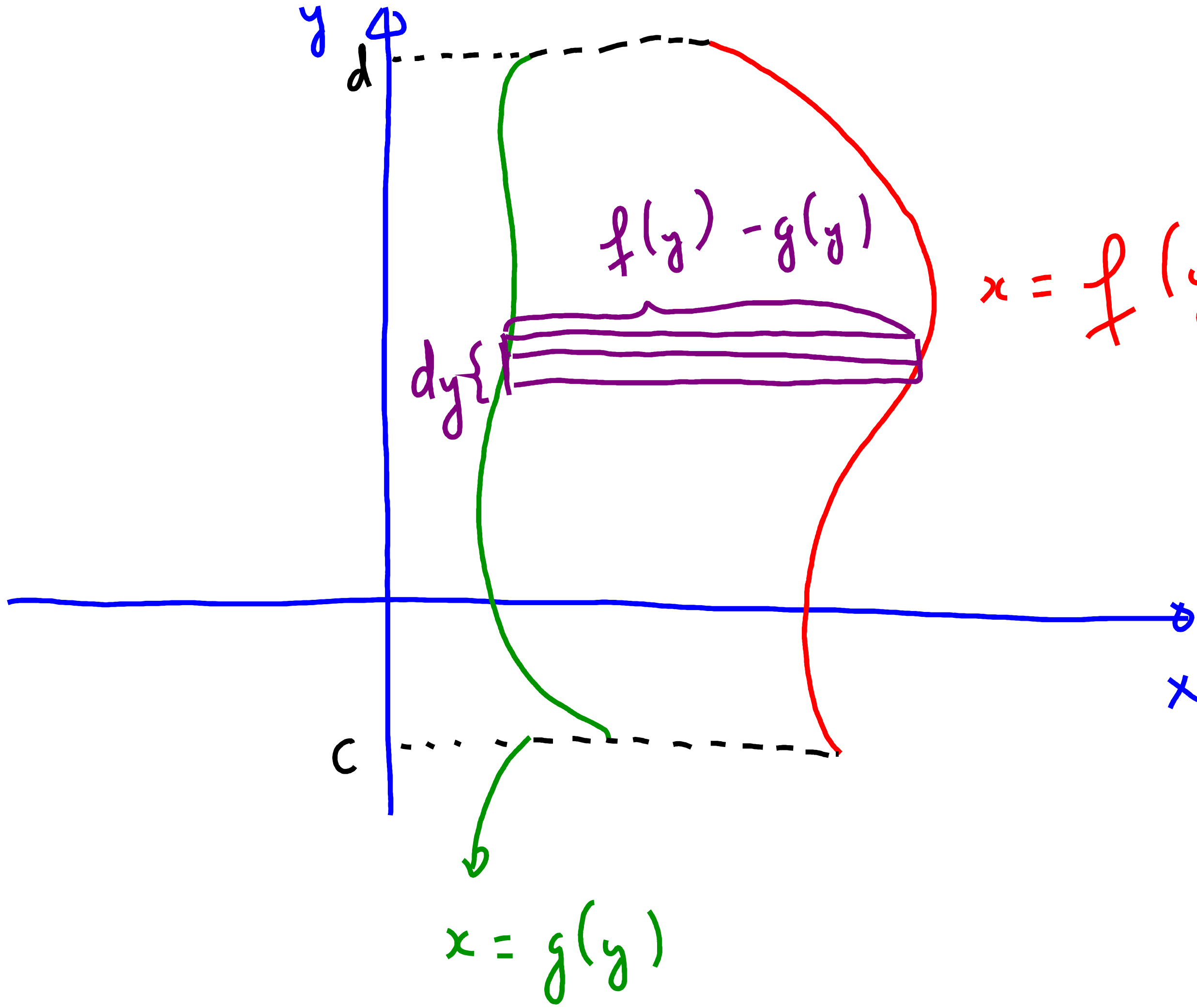
$x(x+1)=0$

$x=-1$; $x=0$

$$\int_{-1}^{0} (-x - x^2)\,dx + \int_{0}^{1} (x - x^2)\,dx$$

$$\left(-\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{-1}^{0} + \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{0}^{1}$$

$$0 - \left(-\frac{1}{2} + \frac{1}{3}\right) + \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{3}}$$



$$\text{Area} = \int_c^d \big(f(y) - g(y) \big)\, dy$$

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$$\begin{array}{r|l} & \boxed{y^2+y-6} \\ y-1 & y^3-7y+6 \\ & -(y^3-y^2) \\ & y^2-7y+6 \\ & -(y^2-y) \\ & -6y+6 \\ & -(-6y+6) \\ \hline & 0 \end{array}$$

$$y^3-7y+6=0$$

$$(y-1)(y^2+y-6)=0$$

$$(y-1)(y-2)(y+3)=0$$

$$\boxed{y=1;\ y=2;\ y=-3}$$

$$\text{Area} = \int_{-3}^{1} \left(y^3-(7y-6)\right)dy = \boxed{\int_{-3}^{1}(y^3-7y+6)\,dy}$$

$$= \cdots$$