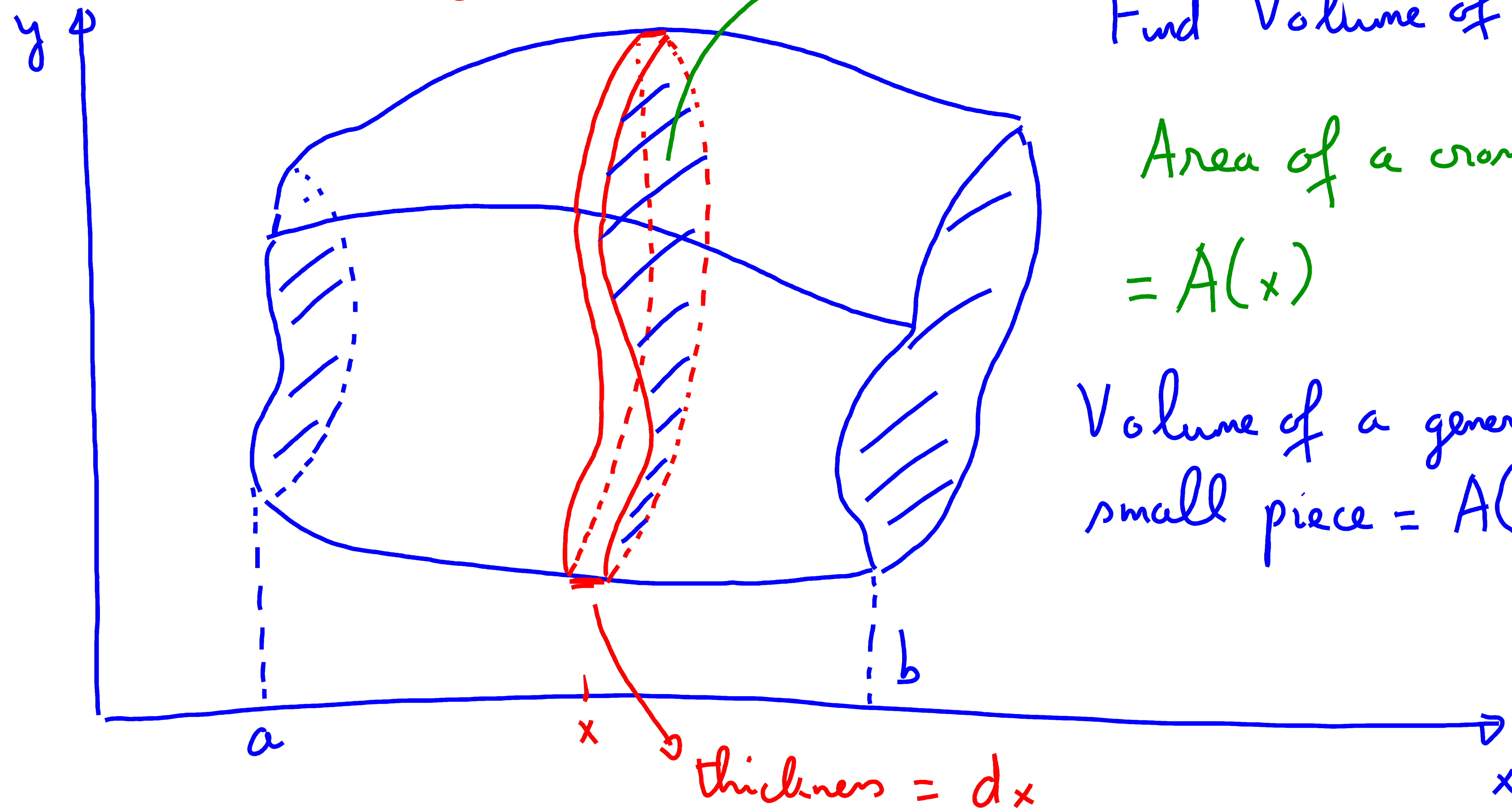
2.2. Volumes by Slicing



Solid S.

Find Volume of S

Area of a cross-section:

$= A(x)$

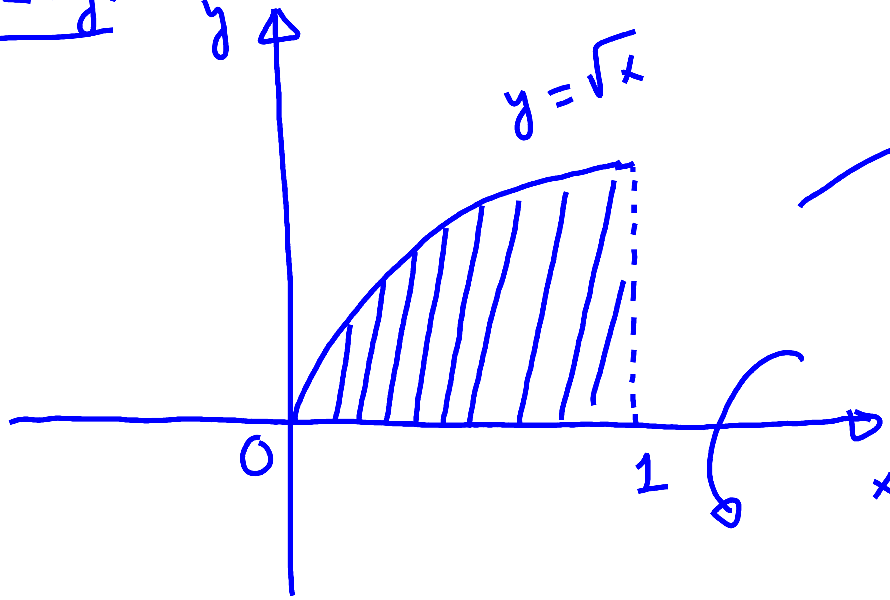
Volume of a generic small piece $= A(x)\,dx$

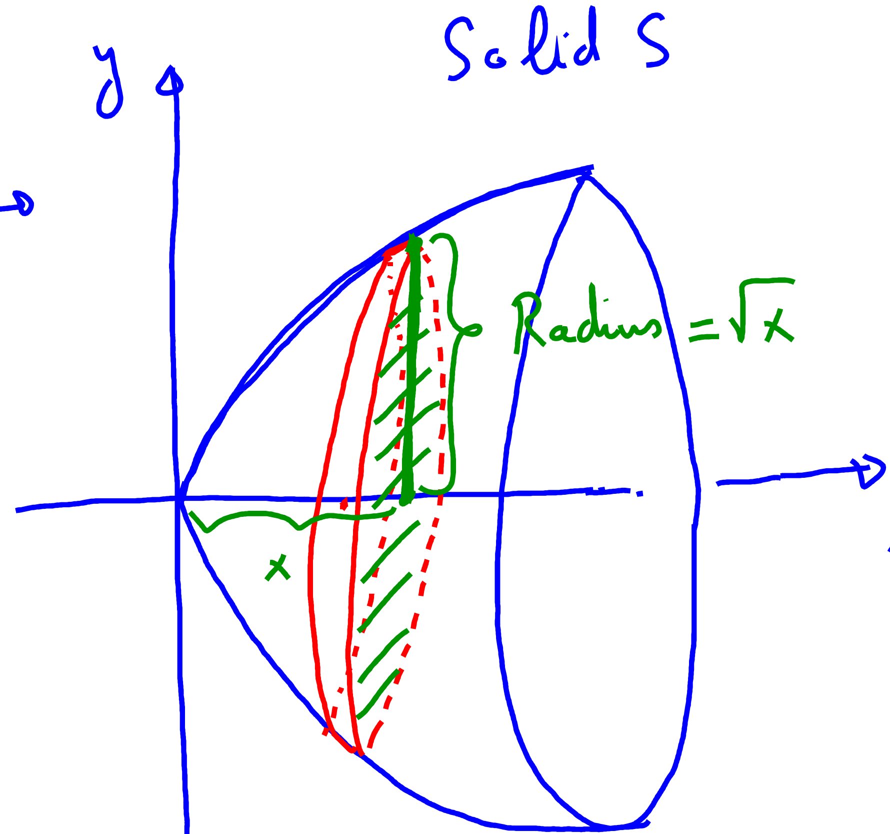
Volume of solid S = Sum of volumes of the pieces:

$$= \boxed{\int_a^b \boxed{A(x)}\,dx}$$

cross-sectional area.

<u>E.g.</u>





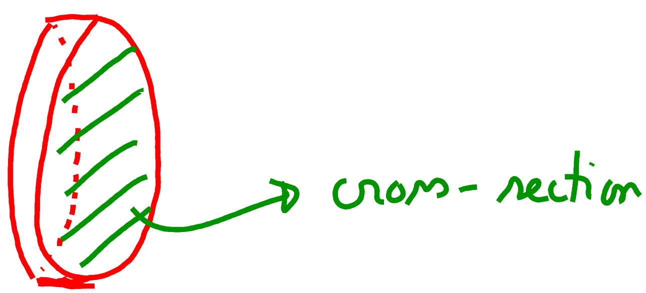
Rotate the region bounded by the graph of $y = \sqrt{x}$; the x-axis from $x = 0$ to $x = 1$ about the x-axis

Find volume of S.

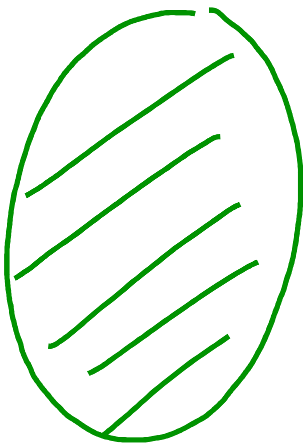
A Slice looks like a disk with some thickness.

thickness $= dx$

cross-sectional area?



A cross section is a disk.



$$\text{Area} = \pi \cdot (\text{Radius})^2$$

$$\text{Radius in our problem} = \sqrt{x}$$

$$\text{Area of a cross-section} = \pi \cdot (\sqrt{x})^2$$

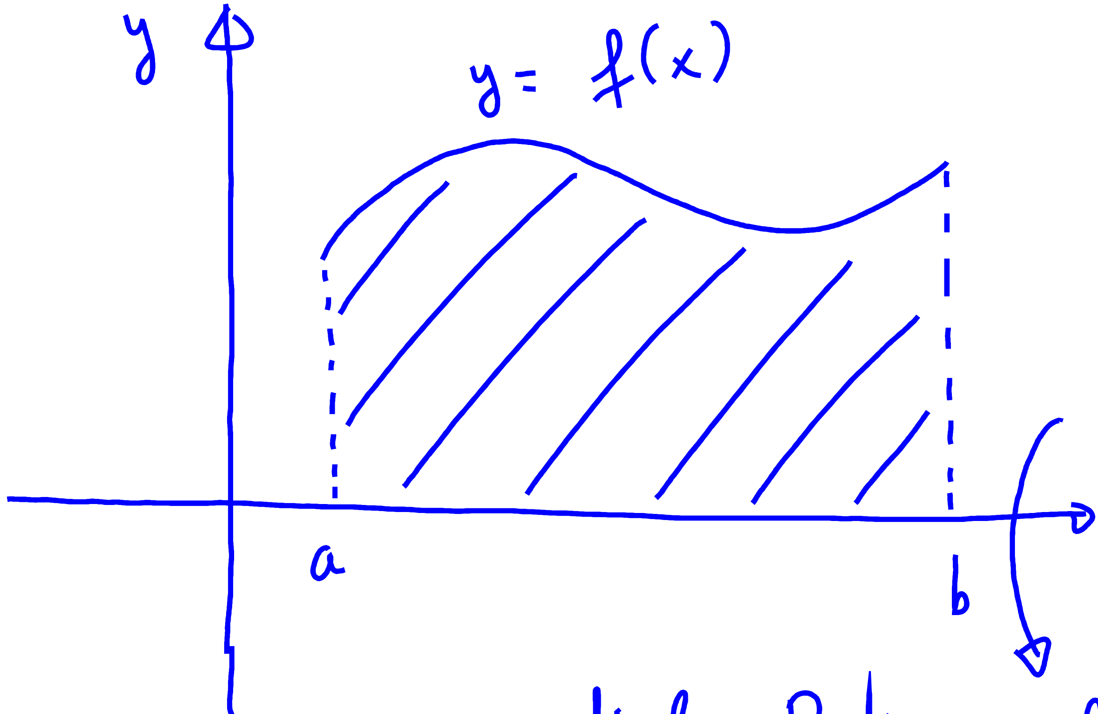
$$= \pi \cdot x$$

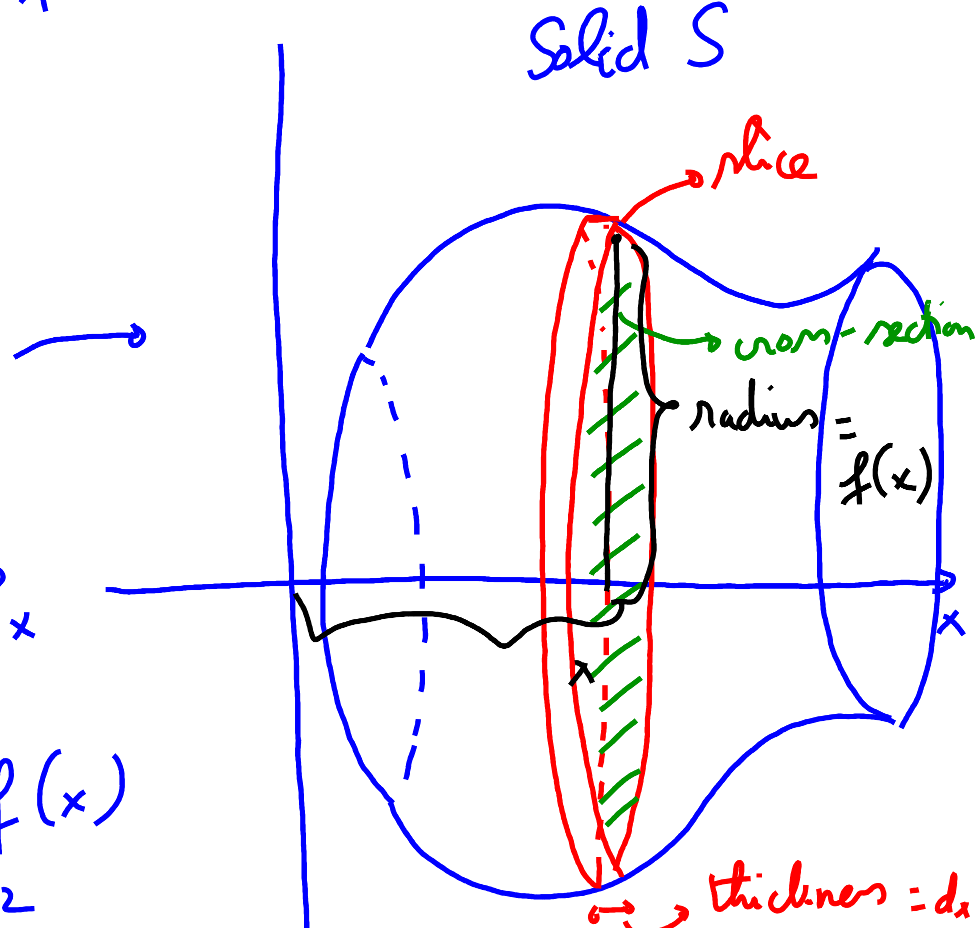
$$\text{Volume of a slice} = \underbrace{\pi x}_{\text{cross-sectional area}} \underbrace{dx}_{\text{thickness}}$$

$$\text{Volume of solid} = \int_0^1 \pi x \, dx$$

$$= \pi \cdot \int_0^1 x \, dx = \pi \cdot \frac{x^2}{2} \Big|_0^1 = \boxed{\frac{\pi}{2}}$$

The disk method to find volume of revolution.



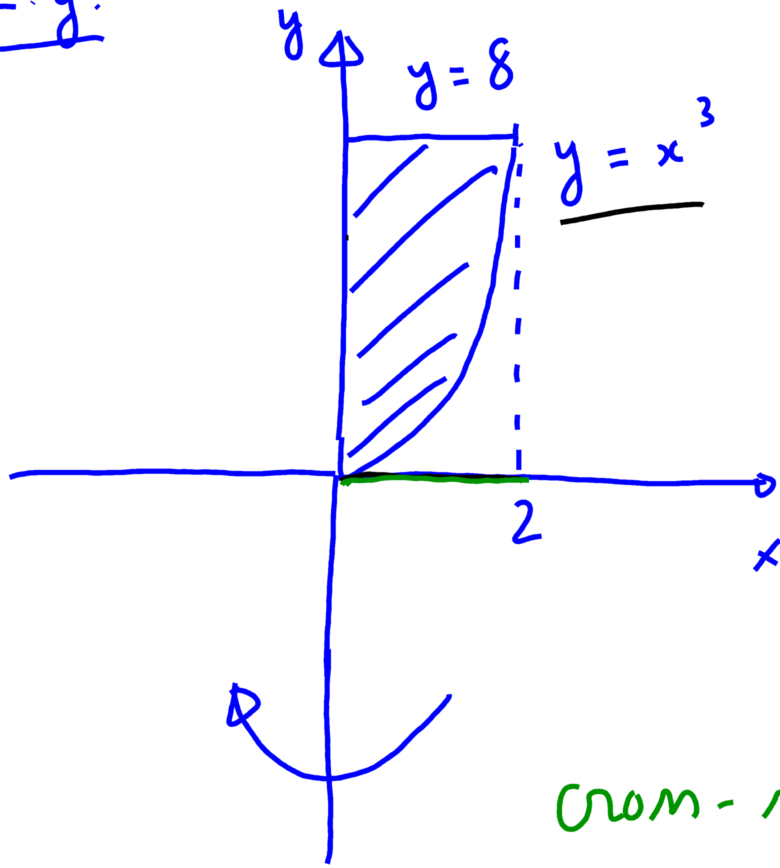


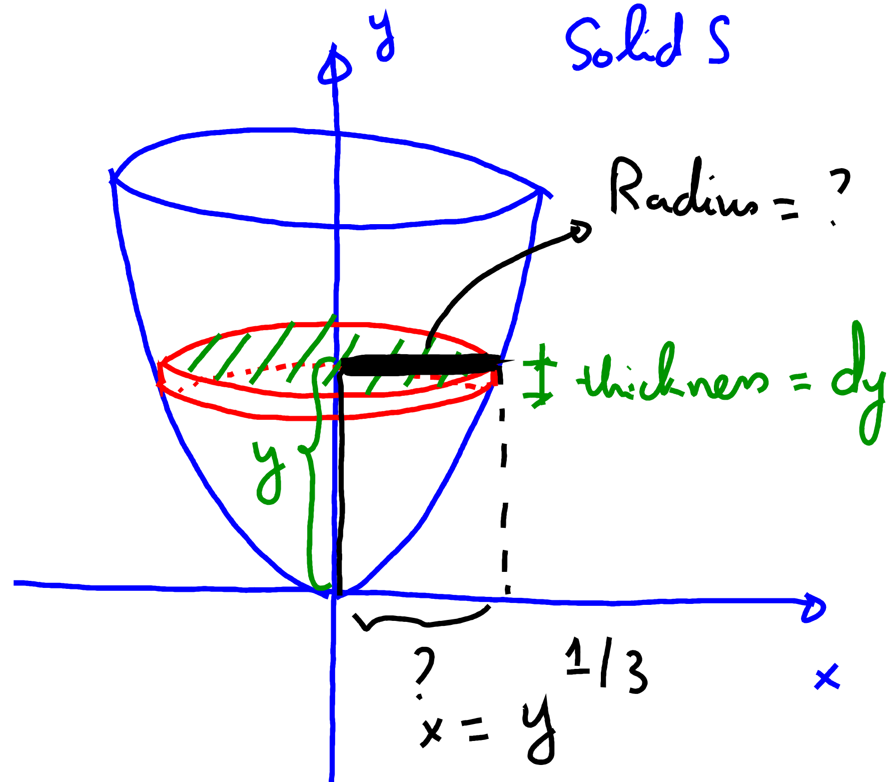
Cross-section is a disk. Radius = f(x)

Cross-sectional area = $\pi \cdot (f(x))^2$

Volume of S = $\boxed{\int_a^b \pi \cdot [f(x)]^2 \, dx}$

E.g.





Volume S = ?

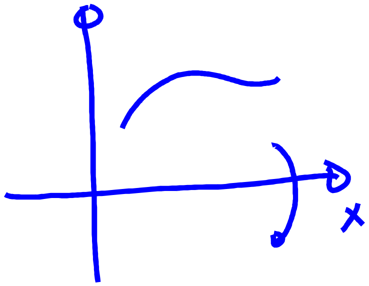
cross-sectional area = ?

$$\pi \cdot \left(y^{1/3}\right)^2 = \pi \cdot y^{2/3}$$

Volume of a slice $= \pi \cdot y^{2/3} \cdot dy$

$$\text{Vol of } S = \int_0^8 \pi \cdot y^{2/3}\, dy = \pi \cdot \int_0^8 y^{2/3}\, dy = \pi \cdot \frac{3y^{5/3}}{5}\Bigg|_0^8 = \boxed{\frac{96\pi}{5}}$$

Disk Method.



Slice along x

- thickness dx
- Radius : y in terms of x

$y = f(x)$

$$\text{Volume} = \int_a^b \pi \cdot [f(x)]^2 \, dx$$

Slice along y

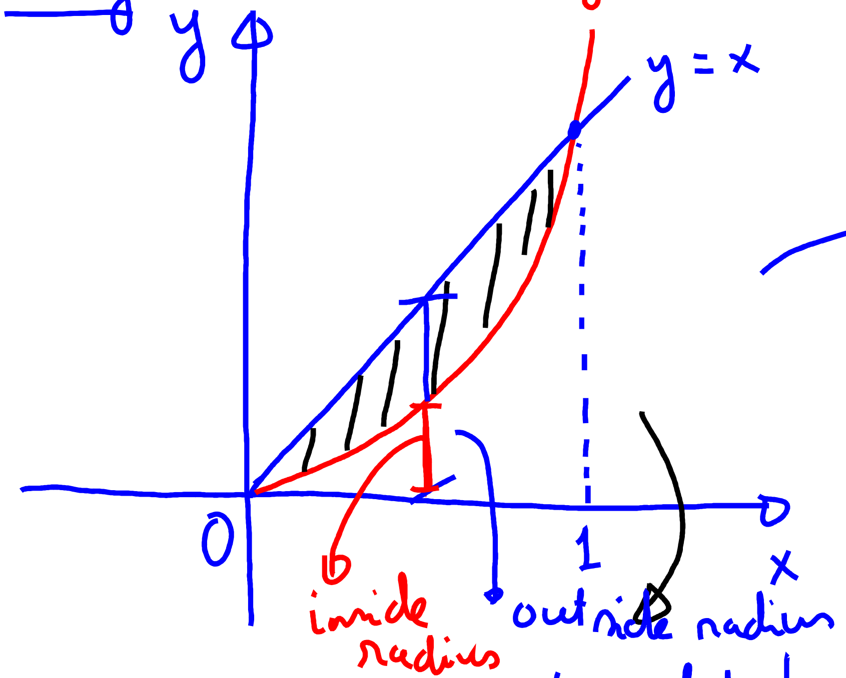
- thickness dy
- Radius : x in terms of y

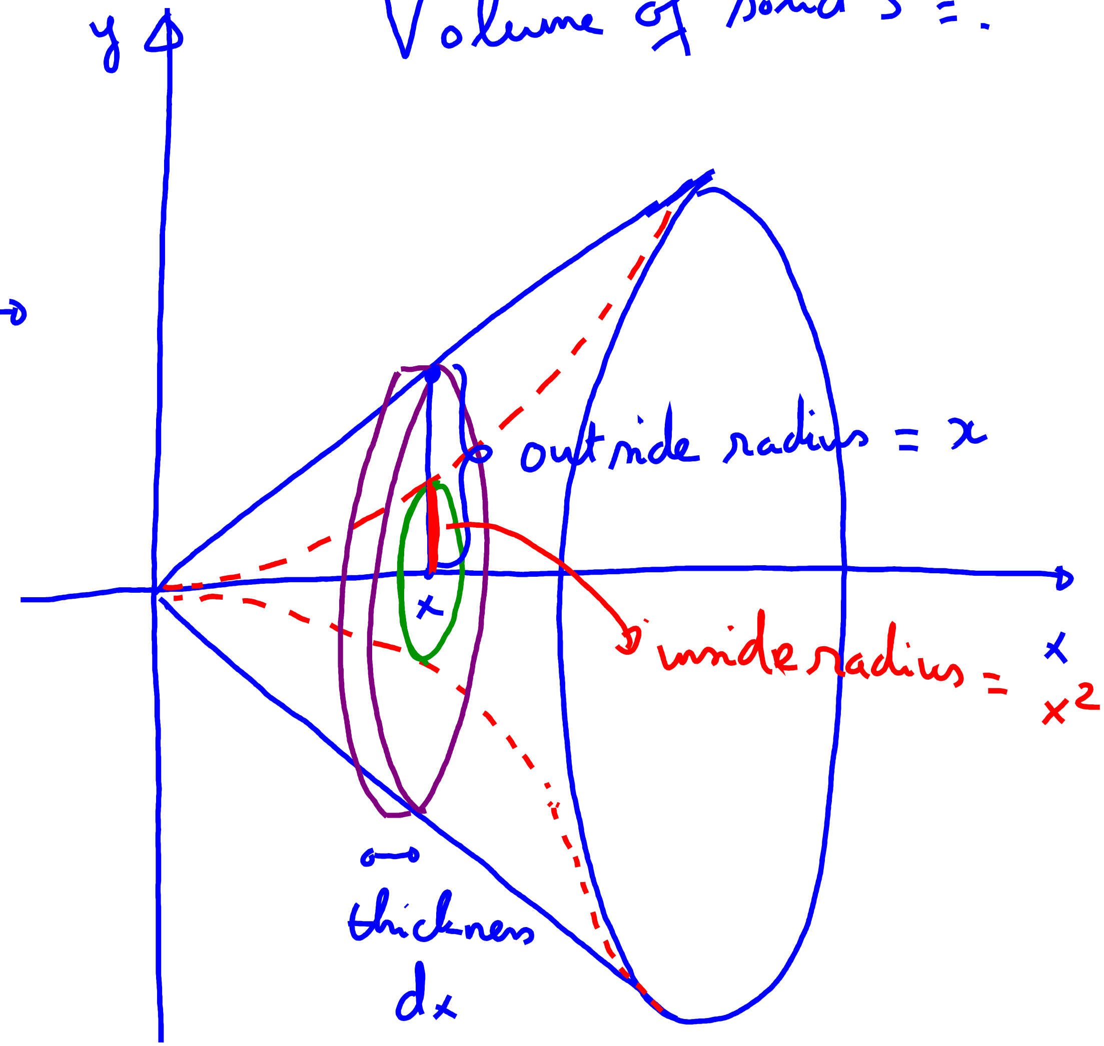
$x = g(y)$

$$\text{Volume} = \int_a^b \pi \cdot [g(y)]^2 \, dy.$$

E.g.

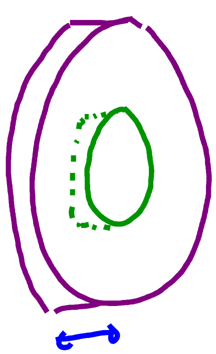
Volume of solid S = ?



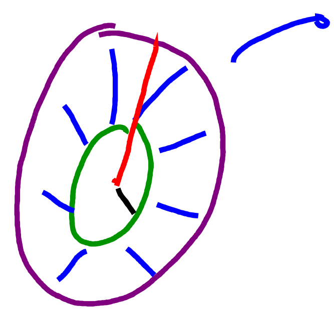


Rotate the region bounded by $y = x^2$; $y = x$ about x-axis.

Slice looks like:



cross-sectional area



cross-section: washer (annular ring)

Area of a cross-section = area of outside circle

$-$ area of inside circle

$= \pi \cdot (\text{outside radius})^2 - \pi \cdot (\text{inside radius})^2$

$= \pi \cdot x^2 - \pi \cdot (x^2)^2$

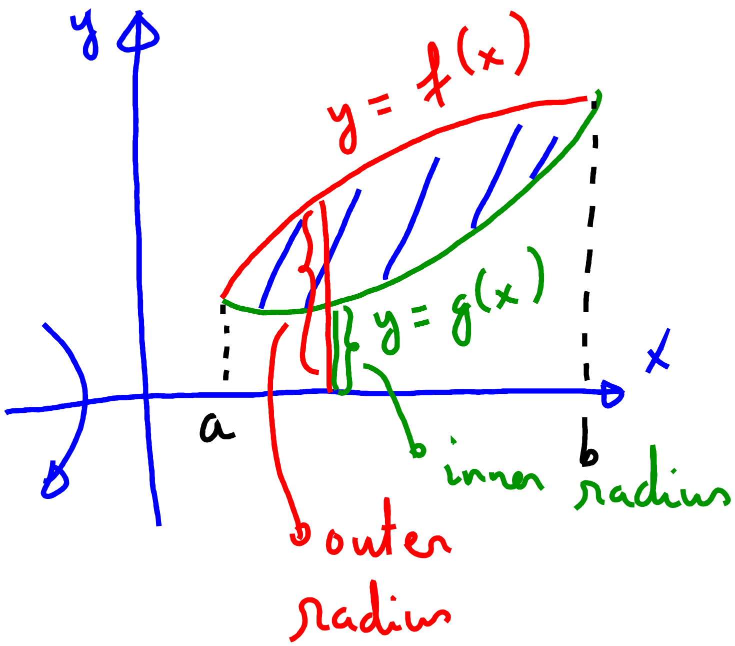
$= \pi x^2 - \pi x^4$

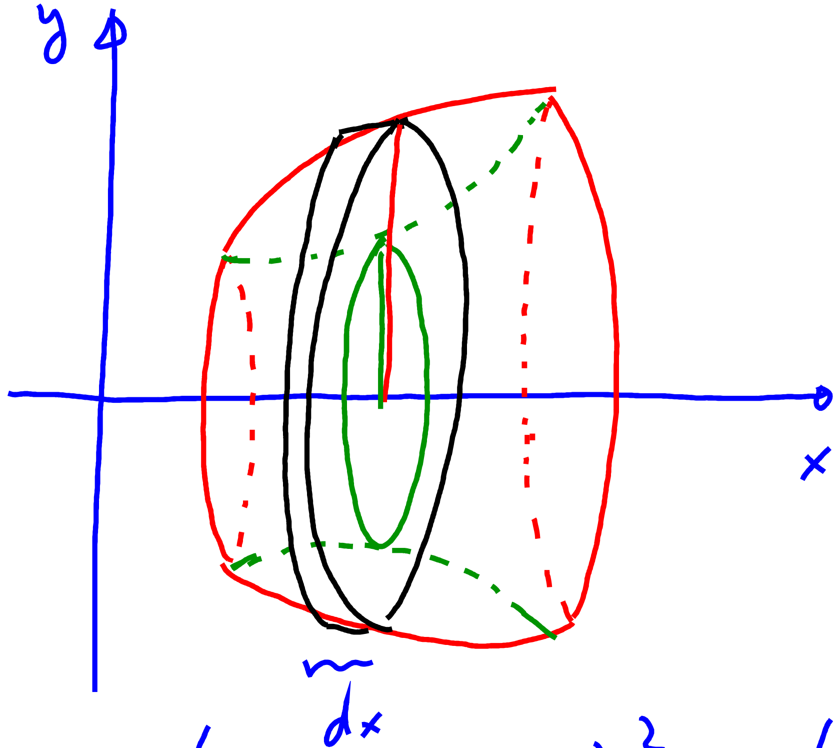
Volume of a slice $= (\pi x^2 - \pi x^4)\,dx$

Volume of S $= \pi \int_0^1 (x^2 - x^4)\,dx = \pi \cdot \left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1$

$= \pi \cdot \left(\frac{1}{3} - \frac{1}{5}\right) = \boxed{\frac{2\pi}{15}}$

Washer Method to find volume of solid of revolution.





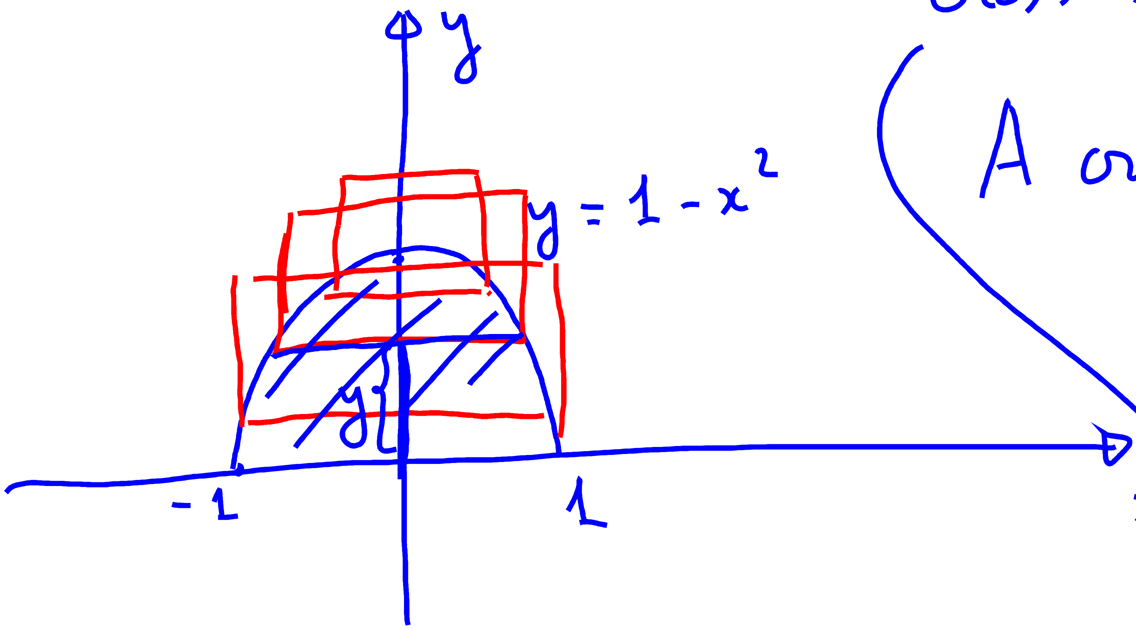
Cross-sectional area $= \pi \cdot (\text{outer radius})^2 - \pi \cdot (\text{inner radius})^2$

$$= \pi \cdot \left([f(x)]^2 - [g(x)]^2 \right)$$

$$\text{Volume} = \pi \int_a^b \left([f(x)]^2 - [g(x)]^2 \right) dx .$$

HW 2.2. - 8

Base of the solid:



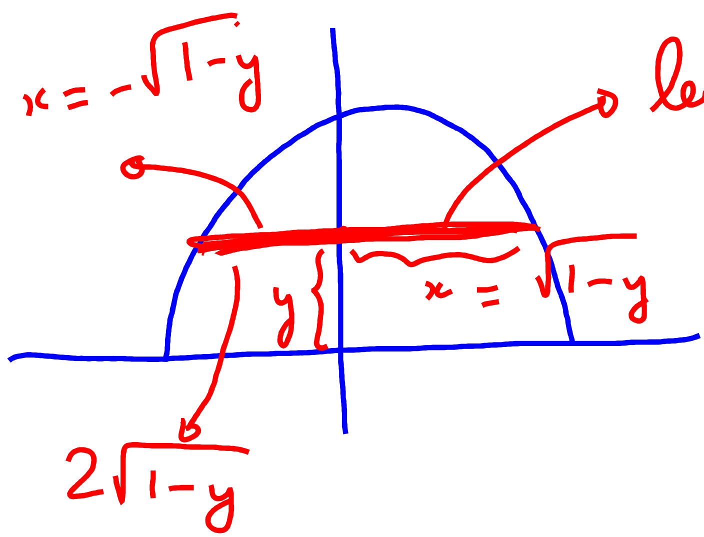
thickness = dy

cross-sectional area =

A cross section is a square

Area of a square = $\left(\boxed{\text{side}}\right)^2$

$$\left(2\sqrt{1-y}\right)^2$$



length of the side of the square

$$y = 1 - x^2 \Rightarrow x^2 = 1 - y; \quad x = \pm\sqrt{1-y}$$

cross sectional area $= \left(2\sqrt{1-y}\right)^2$

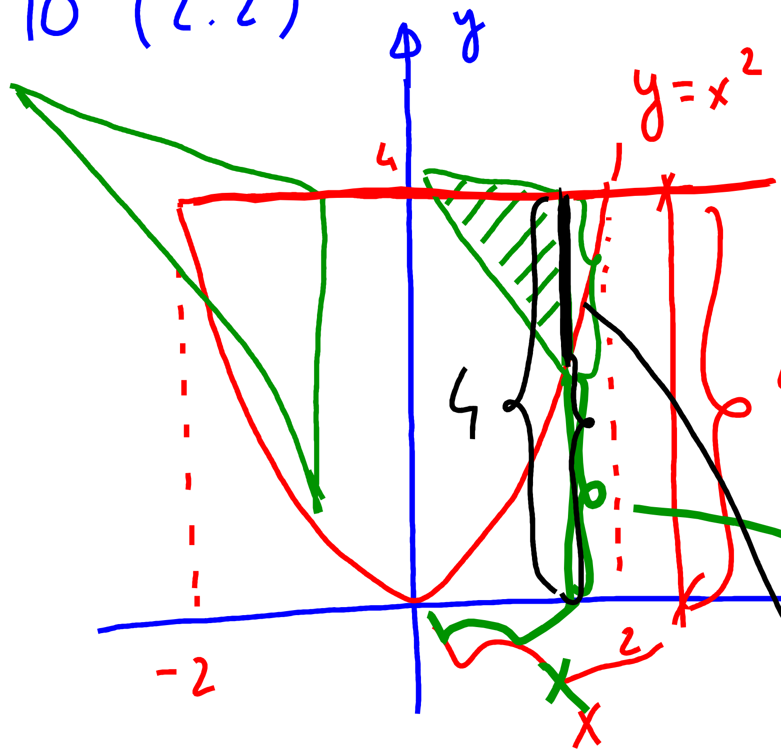
$$\text{Volume} = \int_0^1 \left(2\sqrt{1-y}\right)^2 dy$$

$$= \int_0^1 4 \cdot (1-y)\, dy$$

$$= 4 \cdot \left(y - \frac{y^2}{2}\right)\Big|_0^1$$

$$= 2.$$

#10 (2.2)



Slice it along x-axis.

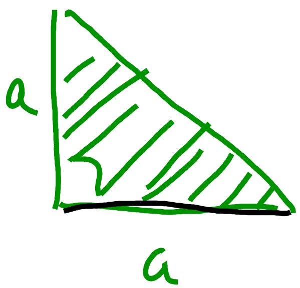
Thickness of a slice : dx

Area of a cross section

4

x^2

$4 - x^2$



$\frac{1}{2} a^2$

Cross-sectional area $= \frac{1}{2} \cdot (4 - x^2)^2$

$$\text{Volume} = \int_{-2}^{2} \underbrace{\frac{1}{2}(4 - x^2)^2}_{\text{cross sectional area}} \underbrace{dx}_{\text{thickness of a slice}}$$